České vysoké učení technické v Praze
Fakulta stavební
Katedra mechaniky

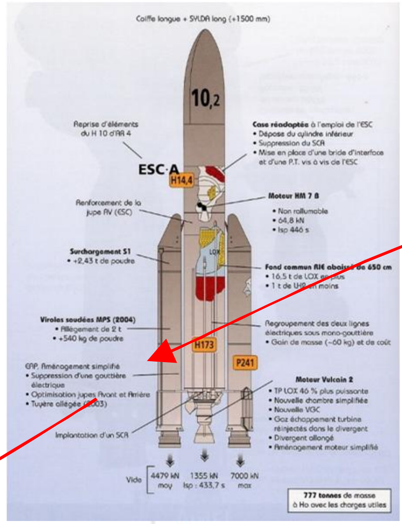
Multi-objective Reliability-based Design Optimization with Meta-models

Matěj Lepš

in cooperation with

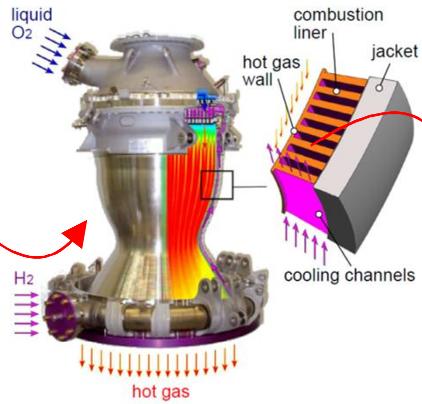
Adéla Pospíšilová and Eva Myšáková

MOTIVATION

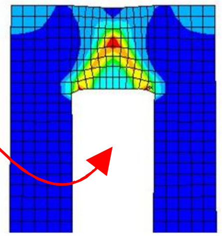


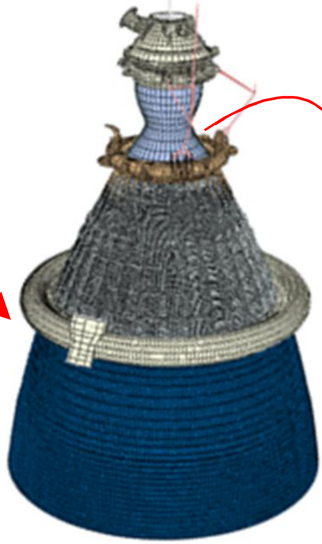
Ariane 5

Thrust chamber



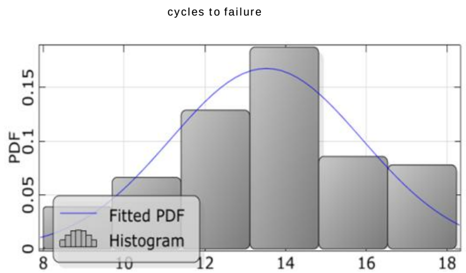
Cooling channel





Engine Vulcain 2

Prediction of resistance

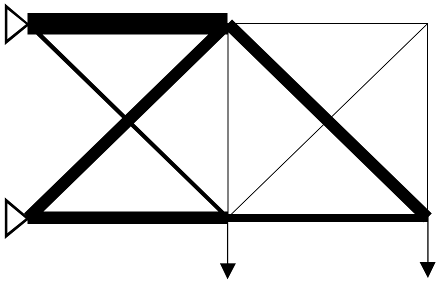


OUTLOOK

- Reliability-based design optimization
- Reliability assessment
- Meta-models
- Computational demands

MOTIVATION:
STRUCTURAL OPTIMIZATION

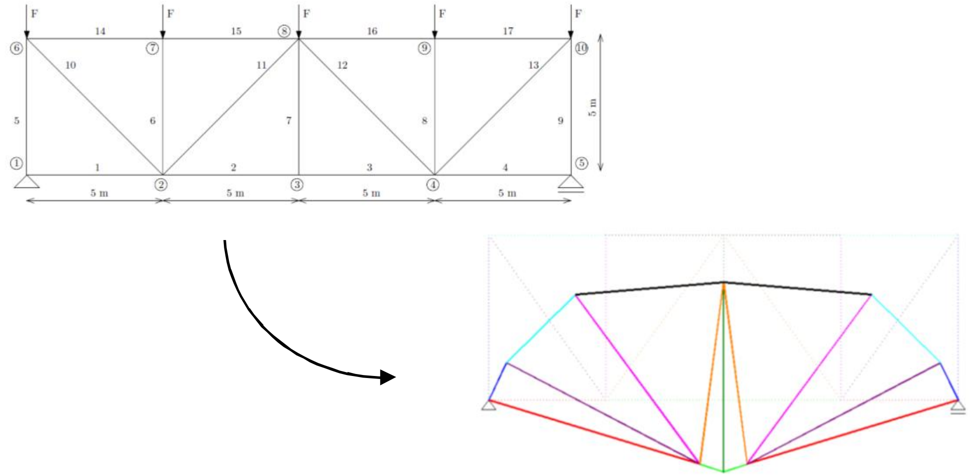
Sizing optimization



Design variables: cross-sections $A_1, \ldots, A_{10}$

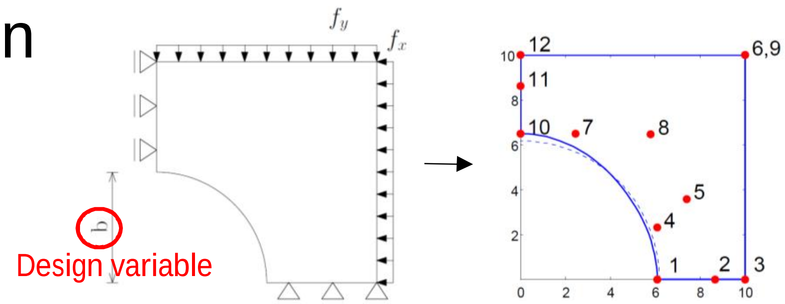
$$\min weight$$
$$\text{s.t.} \max(|disp_i|) \le disp_{max}, i = 1, \ldots, 12$$
$$\max(|\sigma_j|) \le \sigma_{max}, j = 1, \ldots, 10$$

Topology optimization



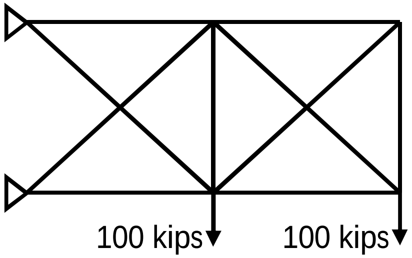
Shape optimization

$$\min \frac{1}{2} f^T u$$
$$\text{s.t.} \quad \boldsymbol{K}u = f$$
$$V = 70 \text{ mm}^2$$



Motivation: Structural Optimization

- Deterministic inputs/outputs



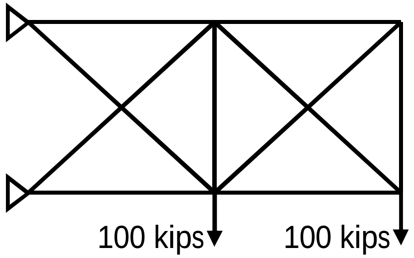
ρ = 0.1 lb/in^3, E = 10^4 ksi

A_{opt} = [31.37; 0.1; 21.48; 15.46; 0.1; 0.1; 2.83; 22.56; 21.86; 0.1] in^2

Weight = 4880.4 lb

MOTIVATION: STRUCTURAL OPTIMIZATION

- Deterministic inputs/outputs



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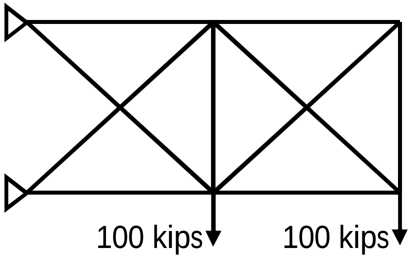
Weight = 4880.4 lb

Uncertainties in material, loading, members, physical model, boundary conditions etc.

Motivation: Structural Optimization

- Deterministic inputs/outputs



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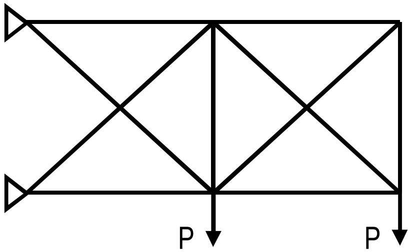
Uncertainties in material, loading, members, physical model, boundary conditions etc.

- Random input/outputs



ρ = 0.1 lb/in^3
E ~ Normal(10^4 , 500) ksi
P ~ [85; 115] kips

$\beta \leq 3$, A ~ Normal (μ_{Ai}, $0.05 \cdot \mu_{Ai}$)
$\mu_{Ai,\,opt}$ = [42.91; 0.1; 29.32; 21.01; 0.1; 0.1; 3.38; 30.81; 29.94; 0.1] in^2
Weight = 6638.0 lb
$\beta = 3 \leq \beta_d = 3$

Motivation: Optimization under Uncertainties

- In real-life constructions uncertainties should be taken into account

- The goal is
  - to provide a design with very small probability of failure that is also economical;

  - or to reduce the system variability to unexpected variations.

MOTIVATION: OPTIMIZATION UNDER UNCERTAINTIES

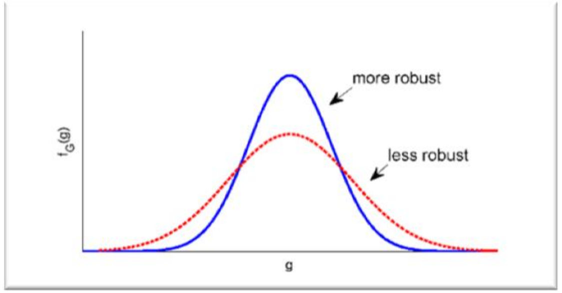
- In real-life constructions uncertainties should be taken into account

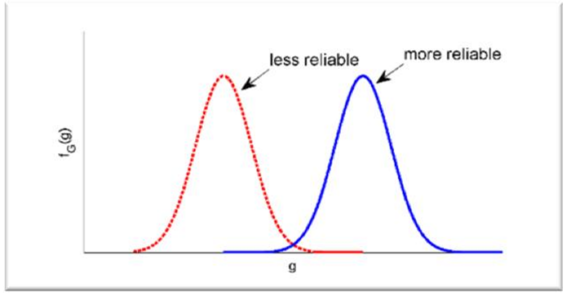
Reliability-based Design Optimization

- The goal is
  - to provide a design with very small probability of failure that is also economical;
  - or to reduce the system variability to unexpected variations.

Robust Design Optimization

Optimization under Uncertainties Problems Classification





Impact of event

Catastrophe

No engineering applications

Reliability-based design and optimization

Performance loss

Robust design and optimization

Reliability is not an issue

Everyday fluctuations

Extreme events

Frequency of event

Motivation:
Reliability-based Design Optimization

$$\min_{\mathbf{d}\in \mathrm{D}} C(\mathbf{x}, \mathbf{d})$$

Minimize costs (e.g. weight of the structure)

$$\max_{\mathbf{d}\in \mathrm{D}} \beta_j(\mathbf{x}, \mathbf{d}), j = 1, \ldots, n_J$$

Maximize safety (Reliability)

$$\left.\begin{aligned} \text{s.t. } & h_i(\mathbf{d}) \leq 0, i = 1, \ldots, n_I \\ & \beta(\mathbf{x}, \mathbf{d}) \leq \beta_j^{tol}, j = 1, \ldots, n_J \end{aligned}\right\}$$

Subject to constraints

$$\Downarrow$$

Multi-Objective Problem

MOTIVATION:
RELIABILITY-BASED DESIG… O…

$$\min_{\mathbf{d}\in D} C(\mathbf{x},\mathbf{d})$$

M

$$\max_{\mathbf{d}\in D} \beta_j(\mathbf{x},\mathbf{d}), j = 1,\ldots,n_J$$

$$\text{s.t.} \quad h_i(\mathbf{d}) \le 0, i = 1,\ldots,n_I$$

$$\beta(\mathbf{x},\mathbf{d}) \le \beta_j^{tol}, j = 1,\ldots,n_J$$

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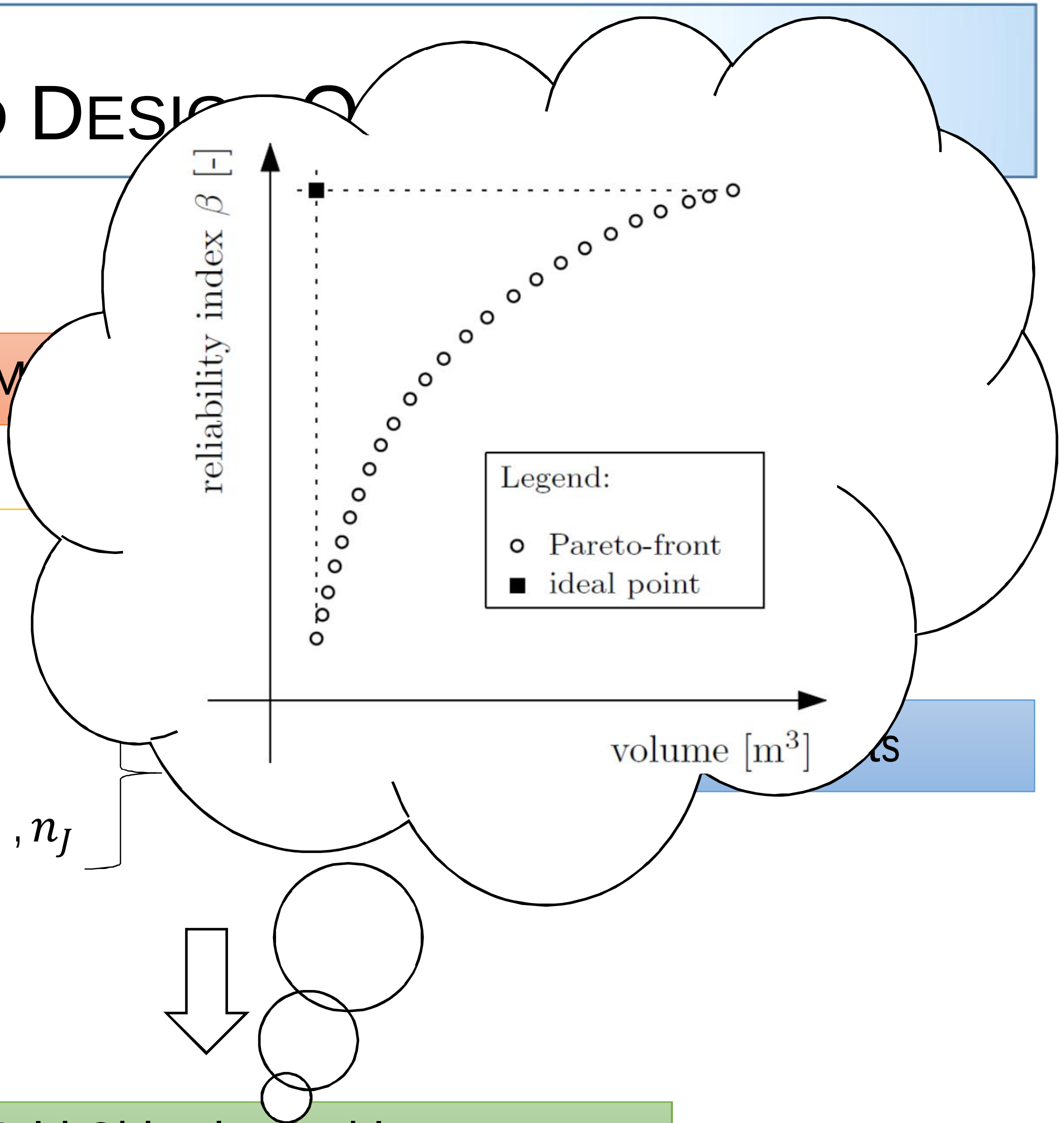
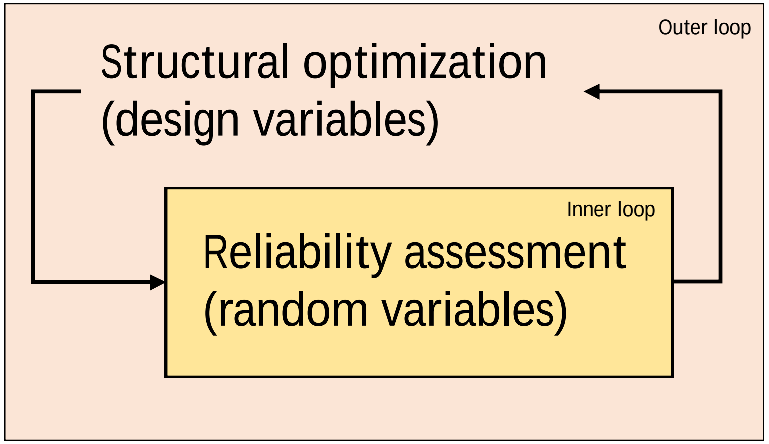


Multi-Objective Problem

Reliability-Based Design Optimization

Double-loop approach

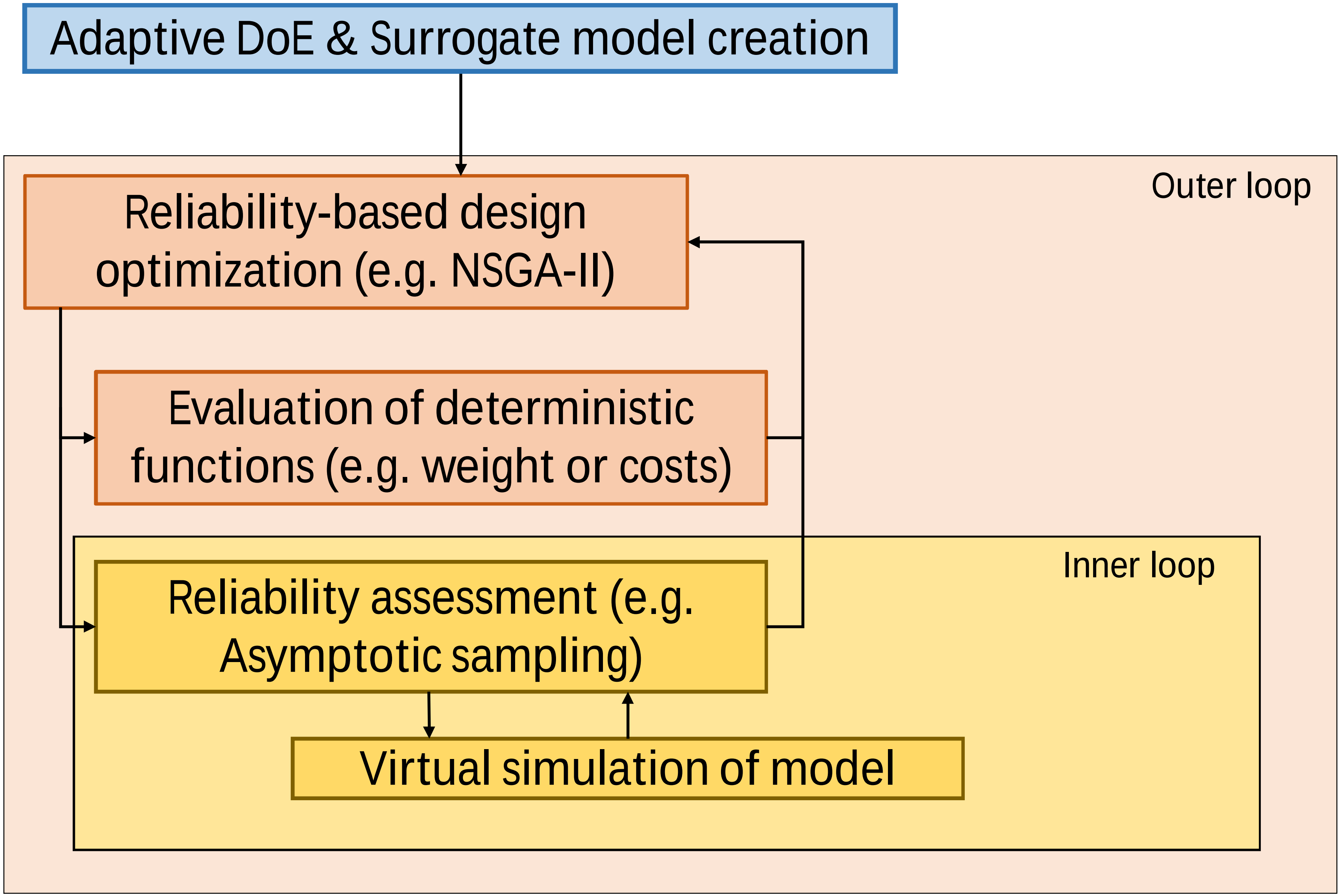


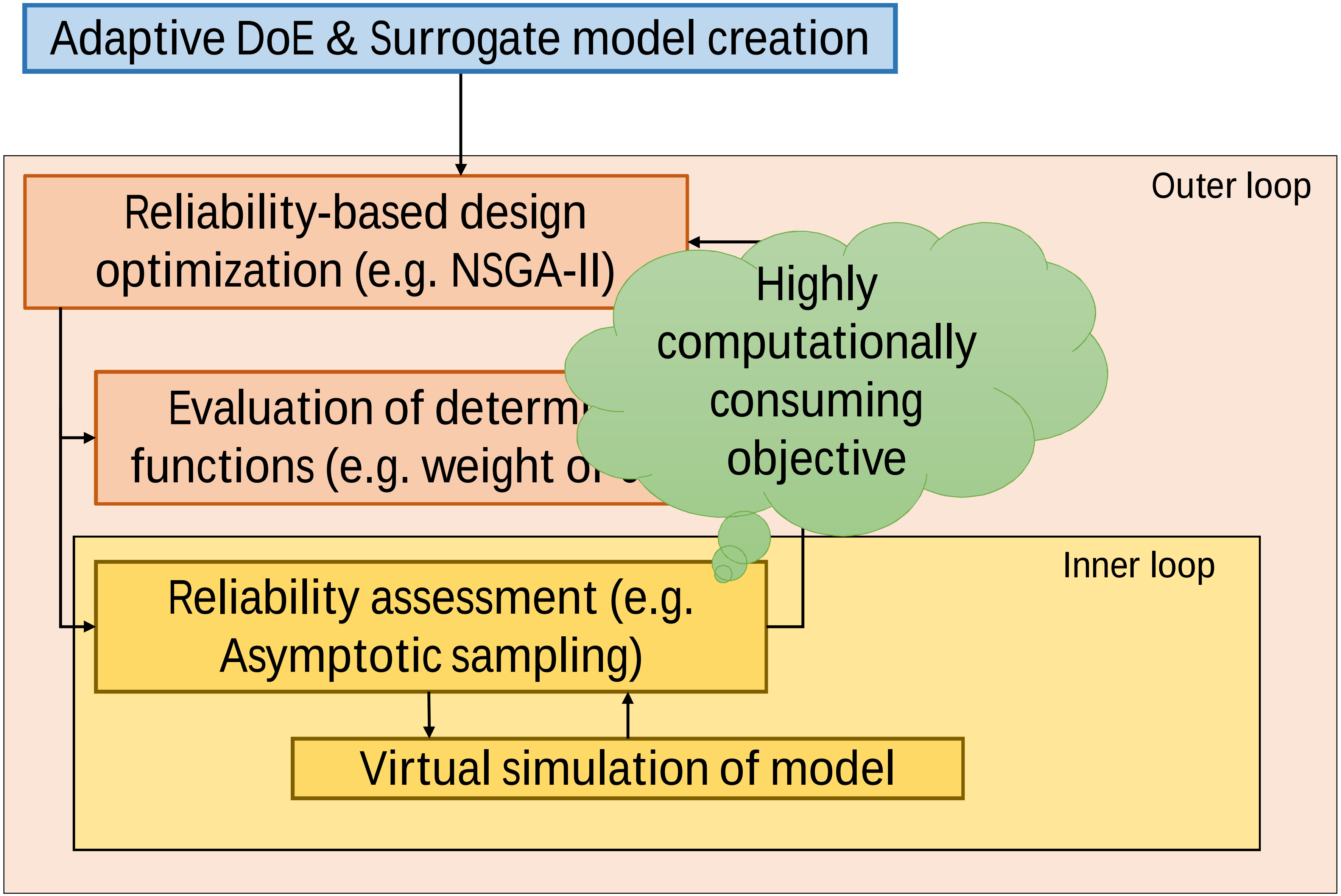
- + Easy to implement
- + Suitable for large number of random variables and failure criteria considering non linear performance functions (sampling-based techniques)
- + Suitable for system reliability constraints

- − Time-consuming

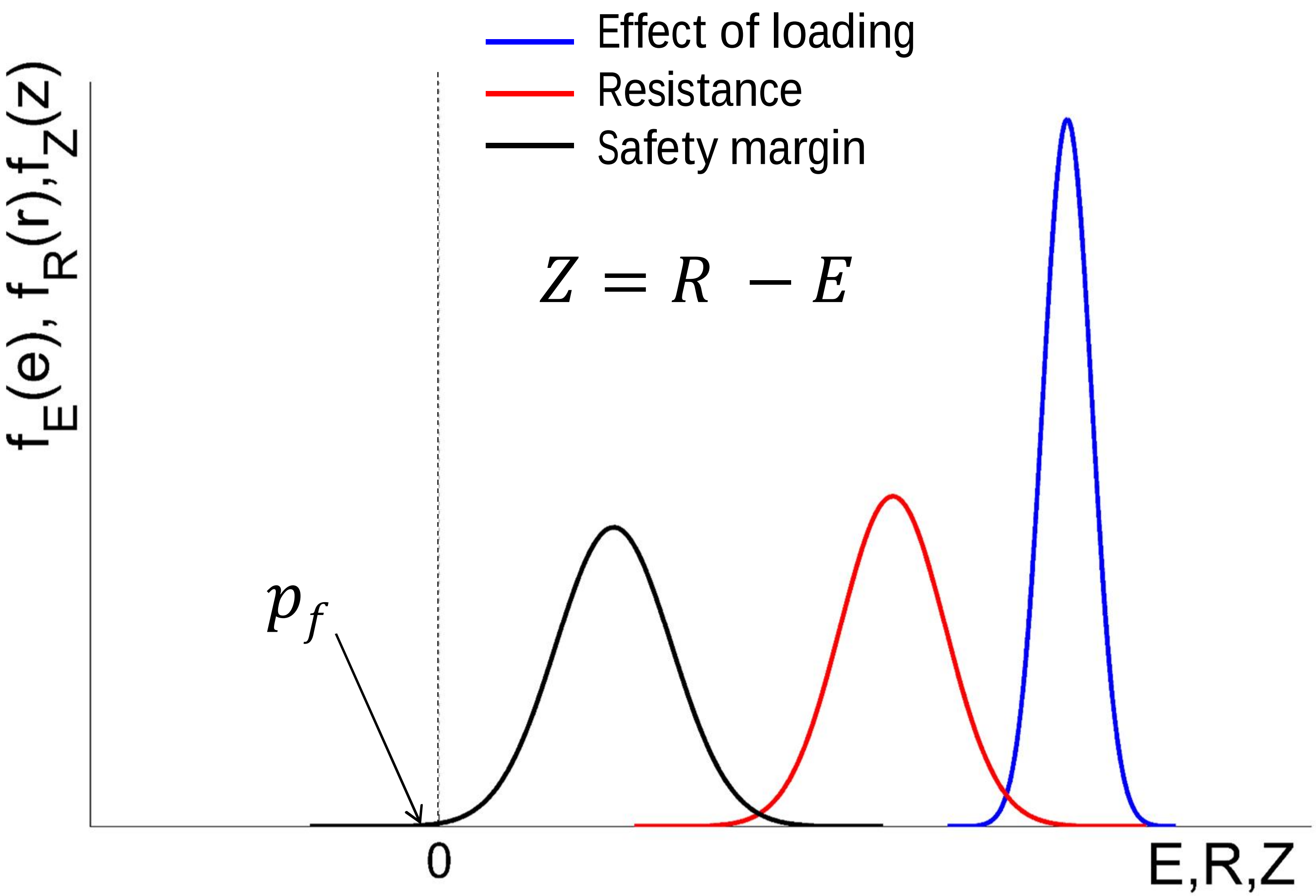
Cost reduction:

- Reliability assessment
  - Approximation techniques based on FORM
  - Advanced sampling-based techniques utilizing meta-models
- Efficient optimization techniques


Adaptive DoE & Surrogate model creation
Outer loop
Reliability-based design optimization (e.g. NSGA-II)
Evaluation of deterministic functions (e.g. weight or costs)
Inner loop
Reliability assessment (e.g. Asymptotic sampling)
Virtual simulation of model


Adaptive DoE & Surrogate model creation
Outer loop
Reliability-based design optimization (e.g. NSGA-II)
Evaluation of determ
functions (e.g. weight o
Highly computationally consuming objective
Inner loop
Reliability assessment (e.g. Asymptotic sampling)
Virtual simulation of model

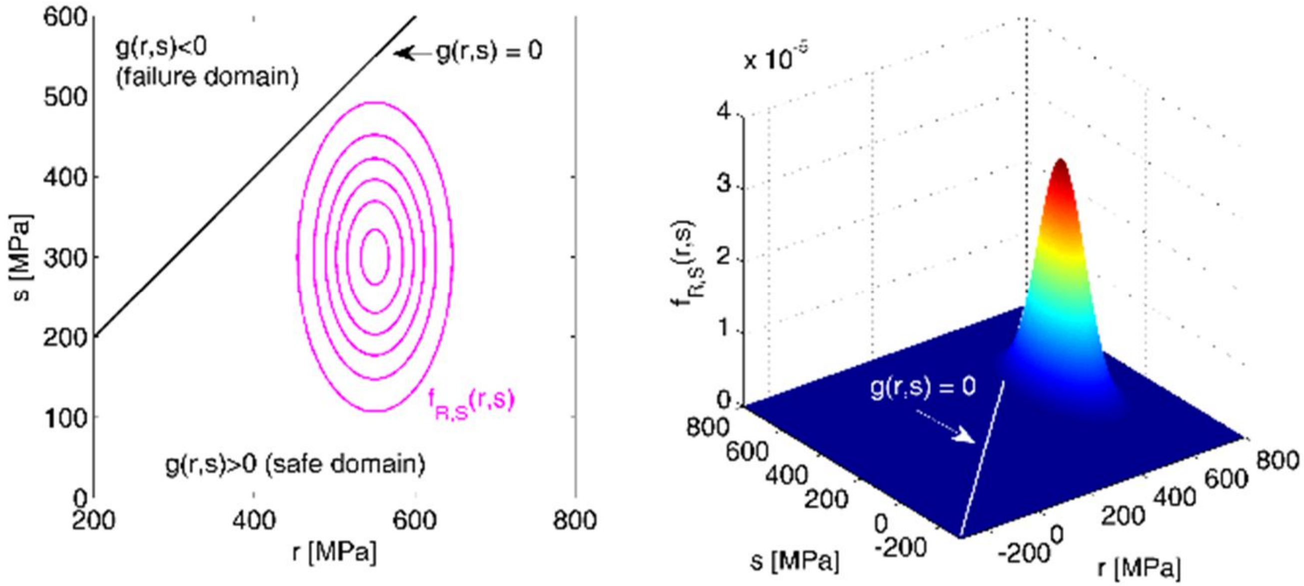
Reliability Assessment



Reliability Assessment

$$p_f = \Pr[g(\mathbf{X}) \leq 0] = \int\limits_{g(\mathbf{X}) \leq 0} \cdots \int f_X(\boldsymbol{x}) \mathrm{d}x$$

- A probability of failure in an n-dimensional space



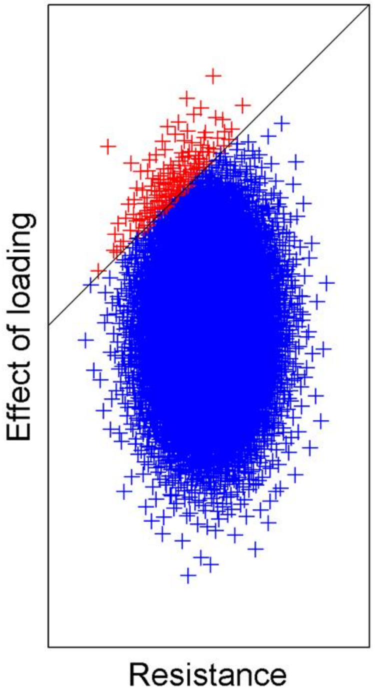
- A generalized reliability index

$$\beta = \Phi^{-1}(1 - p_f)$$

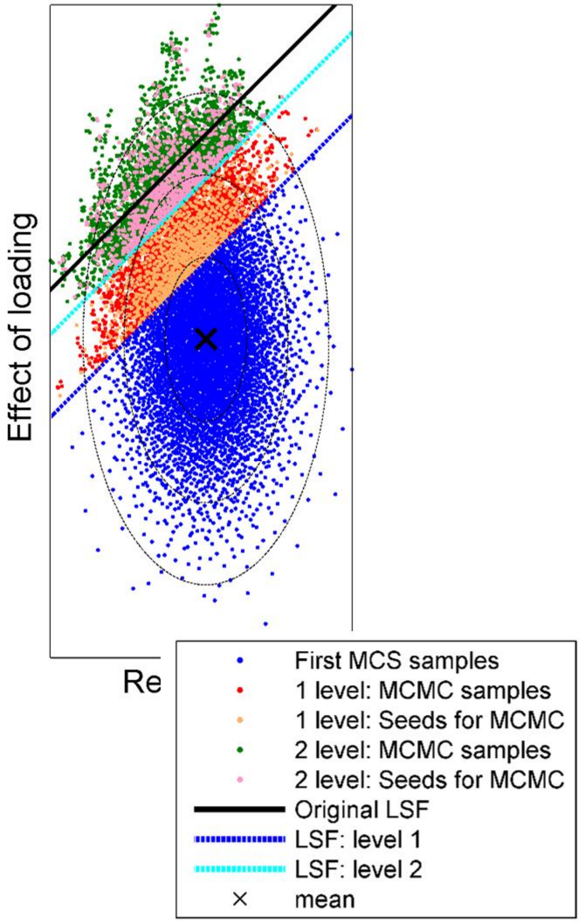
RELIABILITY ASSESSMENT

Examples of simulation techniques

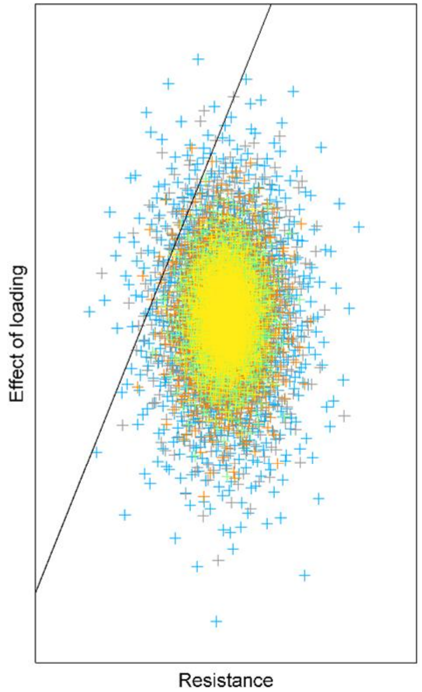
Monte Carlo simulation



Subset Simulation



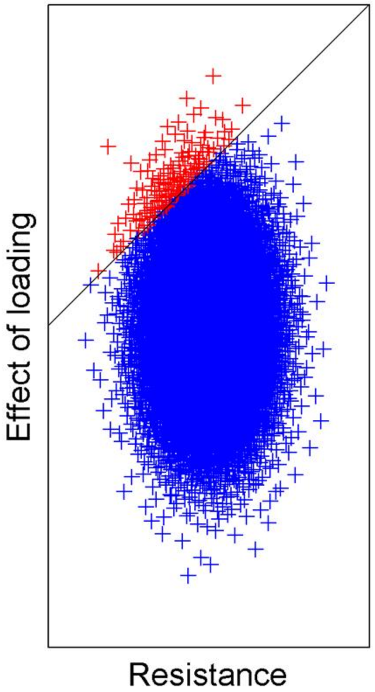
Asymptotic Sampling



RELIABILITY ASSESSMENT

Examples of simulation techniques

Monte Carlo simulation



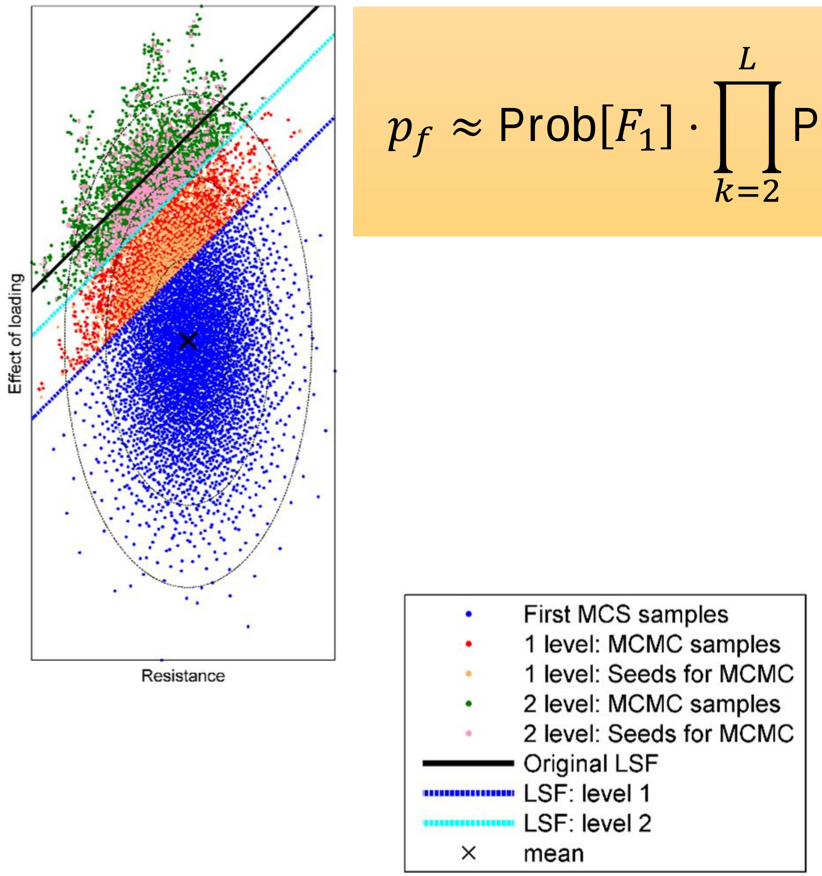
$$p_f \approx \frac{n_f}{n_s}$$

Reliability Assessment

Examples of simulation techniques

Subset Simulation

$$p_f \approx \text{Prob}[F_1] \cdot \prod_{k=2}^{L} \text{Prob}[F_k \,|F_{k-1}]$$

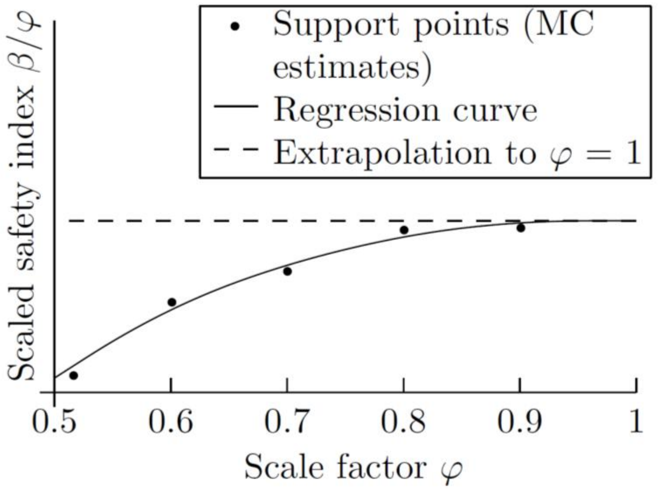


RELIABILITY ASSESSMENT

Examples of simulation techniques

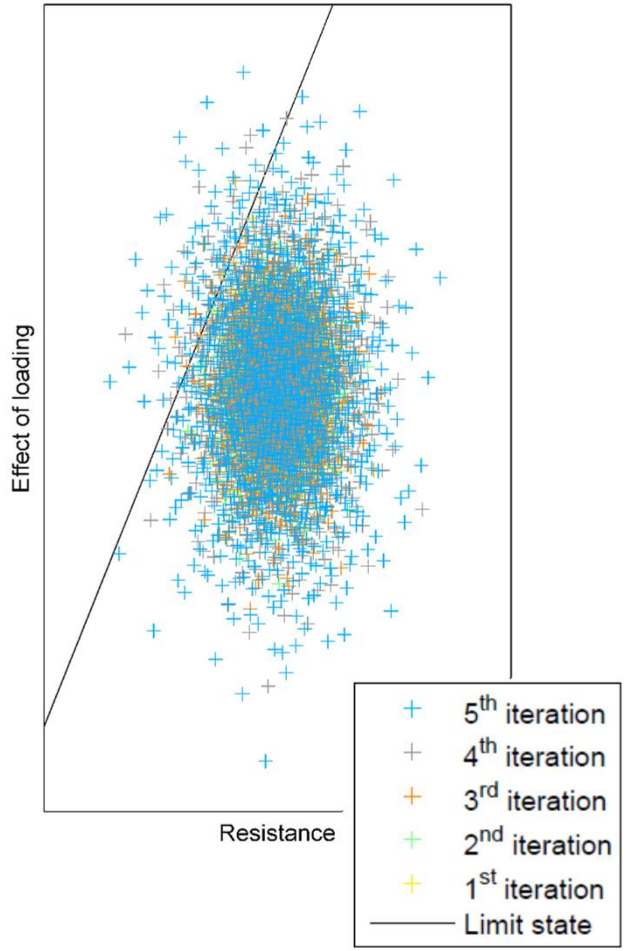
Asymptotic Sampling

$$p_f \approx \Phi(-\beta)$$
$$\beta \approx A + B$$



Regression curve: $\boldsymbol{\beta}_i \approx A\boldsymbol{\varphi}_i + B\boldsymbol{\varphi}_i^{-1}$

$$\varphi_i = {}^{1}/_{\sigma_i}$$



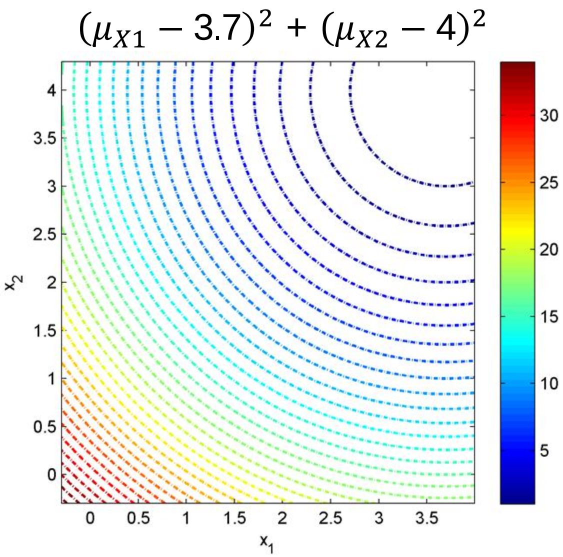
2D Benchmark: 2 Design v., 2 Stochastic v.

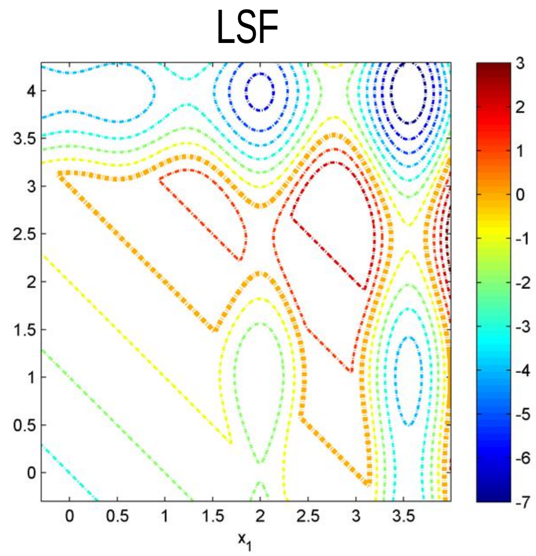
min $(\mu_{X1} - 3.7)^2 + (\mu_{X2} - 4)^2$

max β

considering LSF $\quad \min\begin{pmatrix} -X_1 \sin(4X_1) - 1.1X_2 \sin(2X_2) \\ X_1 + X_2 - 3 \end{pmatrix}$

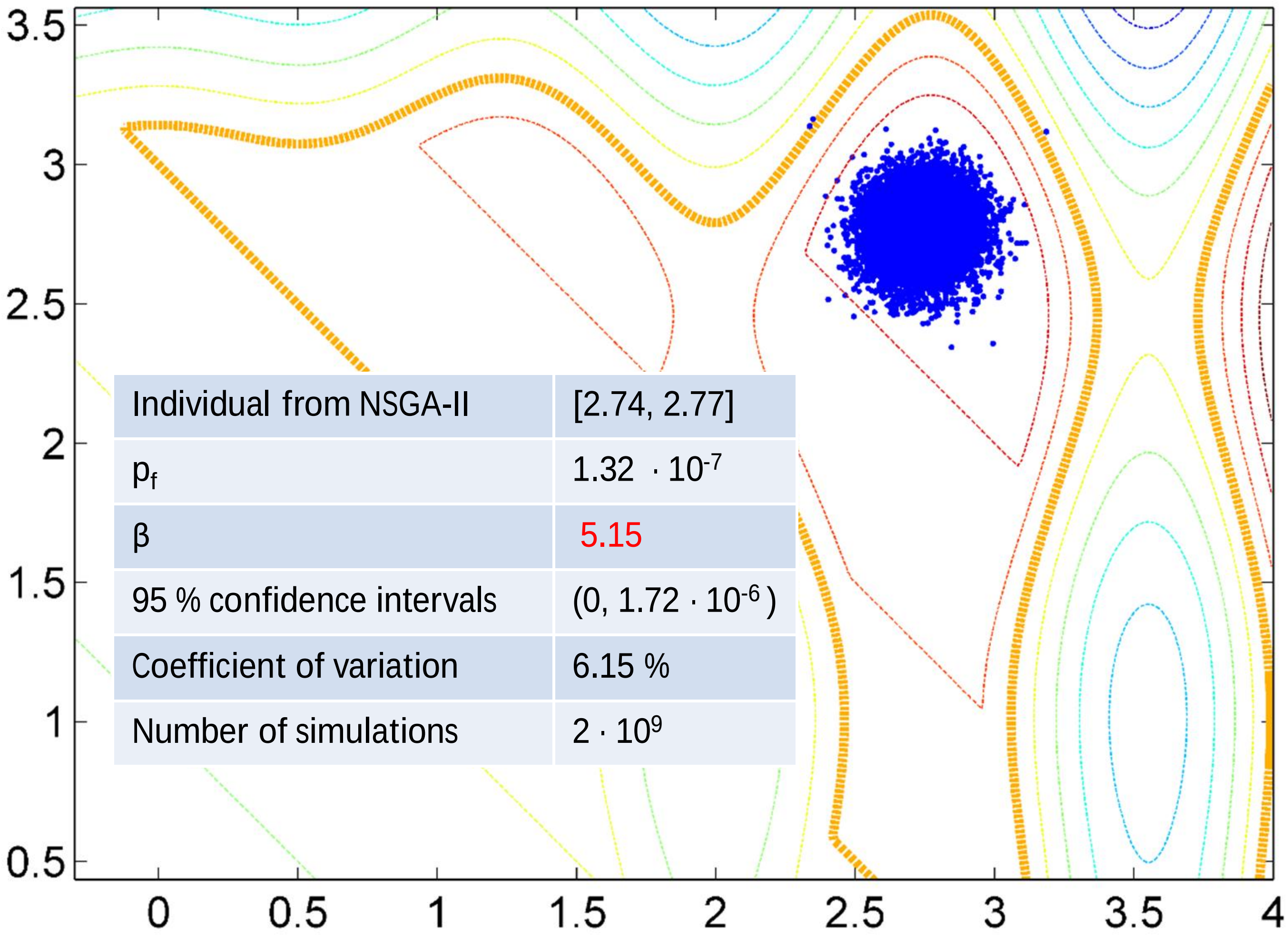
$0 \le \mu_{X1} \le 3.7, 0 \le \mu_{X2} \le 4$





orange contour is for $g(\mathbf{X}) = 0$

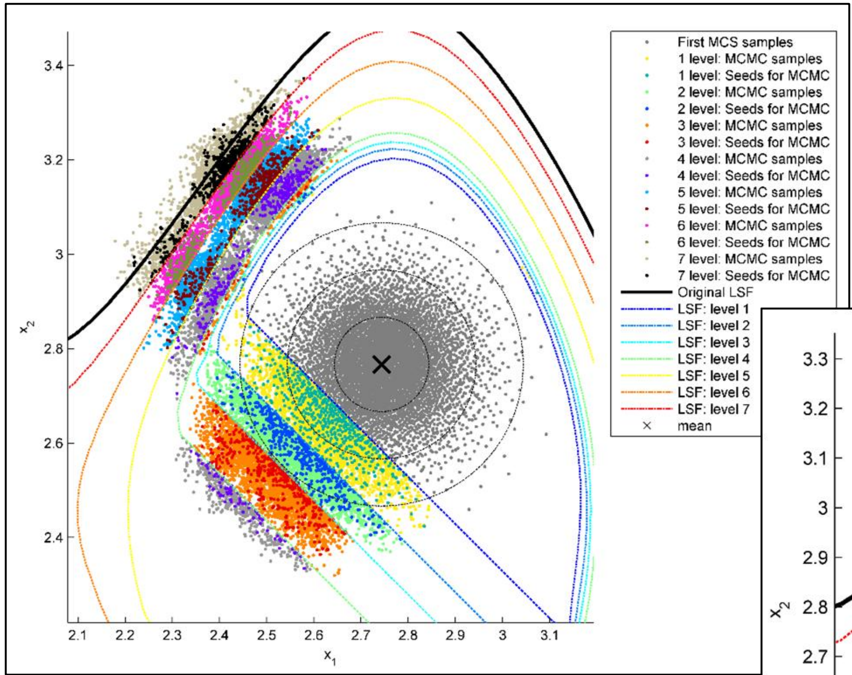
2D BENCHMARK: MONTE CARLO ON ARBITRARY POINT



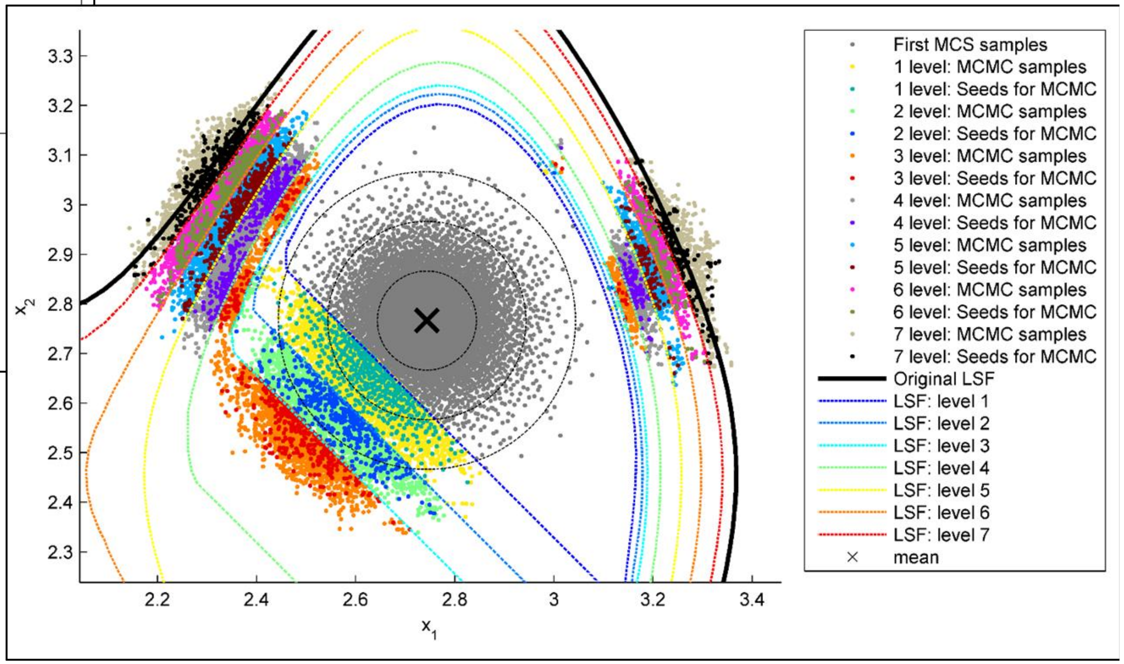
Individual from NSGA-II	[2.74, 2.77]
p_f	$1.32 \cdot 10^{-7}$
β	5.15
95 % confidence intervals	$(0, 1.72 \cdot 10^{-6})$
Coefficient of variation	6.15 %
Number of simulations	$2 \cdot 10^{9}$

2D Benchmark: Subset Simulation on Arbitrary Point

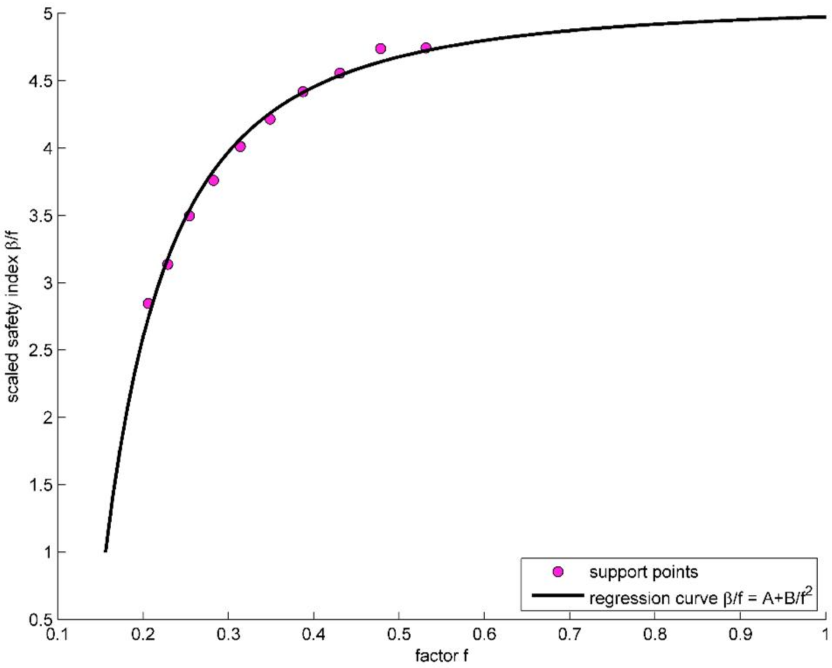
β = 5.533



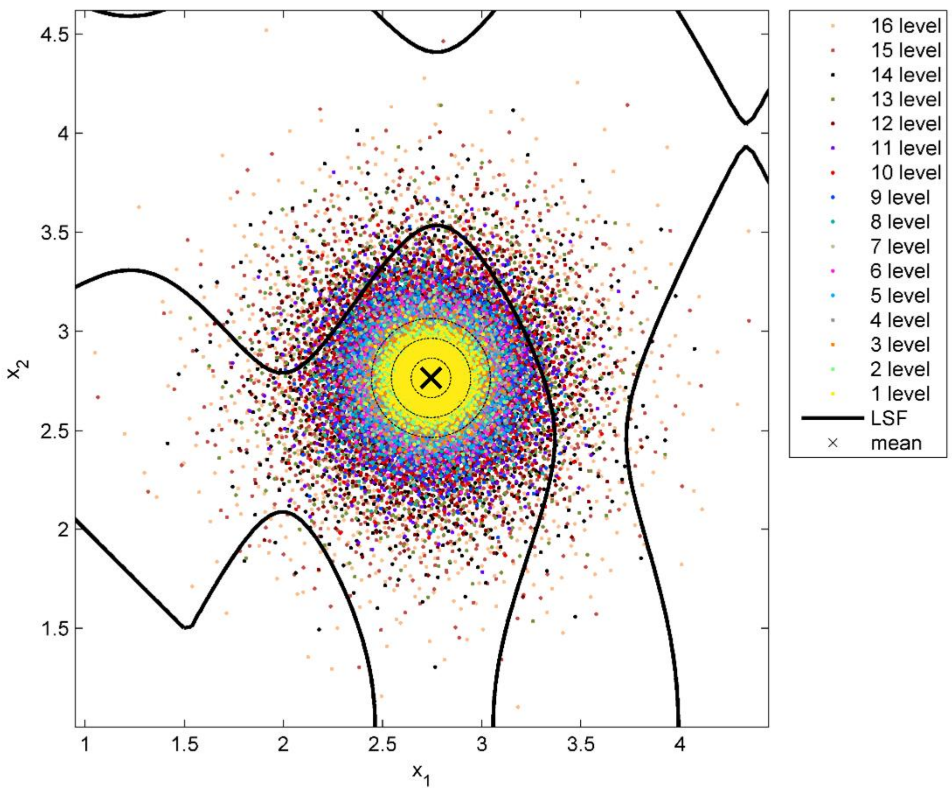
β = 5.4427



2D Benchmark: Asymptotic Sampling on Arbitrary Point



$\beta(1) = 4.974$

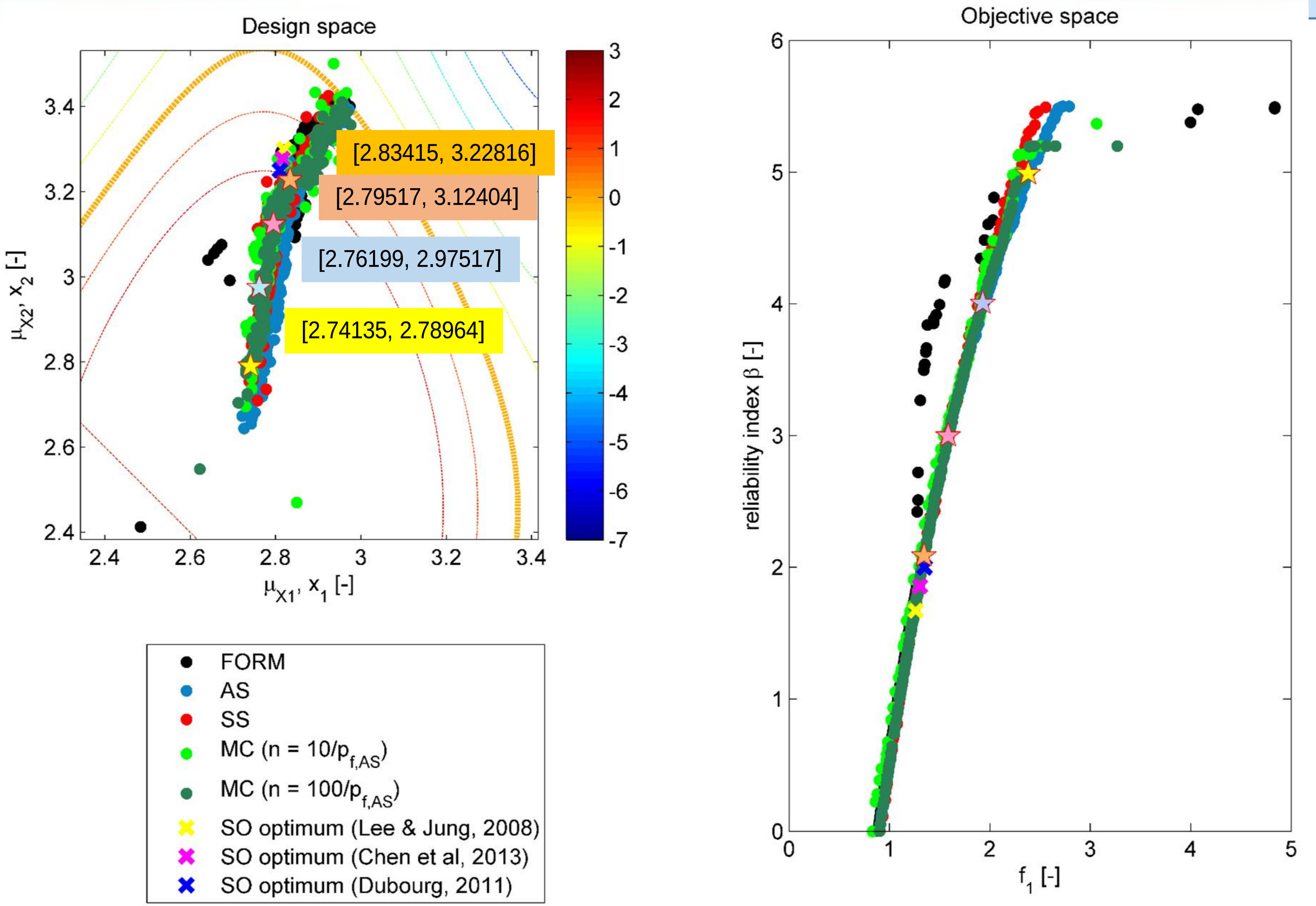


2D Benchmark: RBDO Comparison

Tab: Number of model evaluations for whole RBDO procedure with different reliability assessment methods (1 run: 100 individuals, 20 generations, NSGA-II)

Reliability assessment method	Number of model evaluations
First Order Reliability Method	$6.10 \cdot 10^{4}$
Asymptotic Sampling	$4.15 \cdot 10^{7}$
Subset Simulation	$4.49 \cdot 10^{7}$
Monte Carlo simulation (10/$p_{f,AS}$),ca 30 % CoV	$1.93 \cdot 10^{11}$
Monte Carlo simulation (100/$p_{f,AS}$), ca 10 % CoV	$1.13 \cdot 10^{12}$

2D BENCHMARK: RBDO COMPARISON



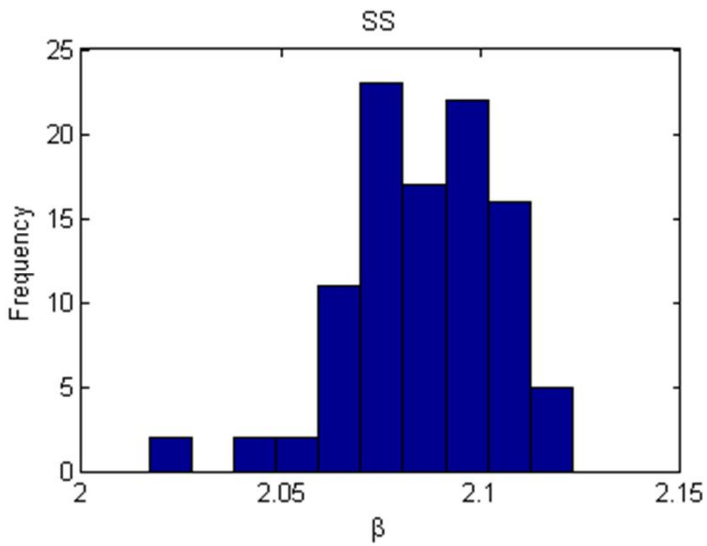
2D BENCHMARK: RBDO COMPARISON

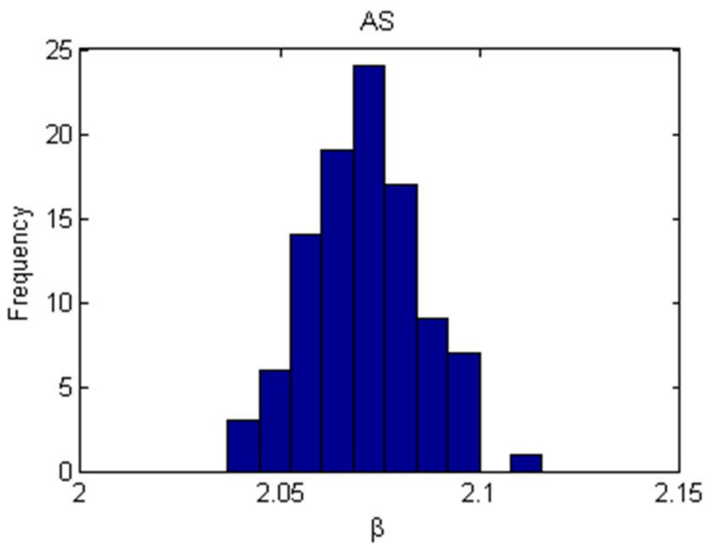
[2.83415, 3.22816]

100 run	SS	AS
Min β	2.0355	2.0430
Max β	2.1301	2.1071
Mean β	2.0837	2.0740
Standard deviation β	0.0189	0.0132
Coeff. of variation β	0.91 %	0.64 %
Number of samples	12,000	12,000

Single run with MC:

β	2.0902
CoV MC	2.32 %
Number of sim.	10,000





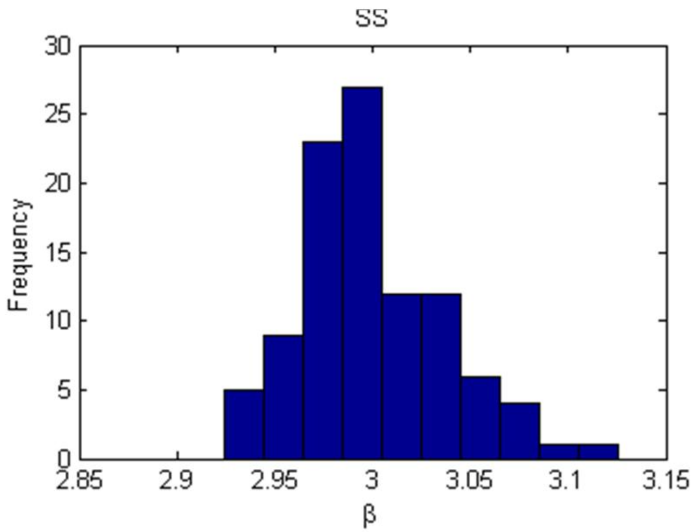
2D Benchmark: RBDO Comparison

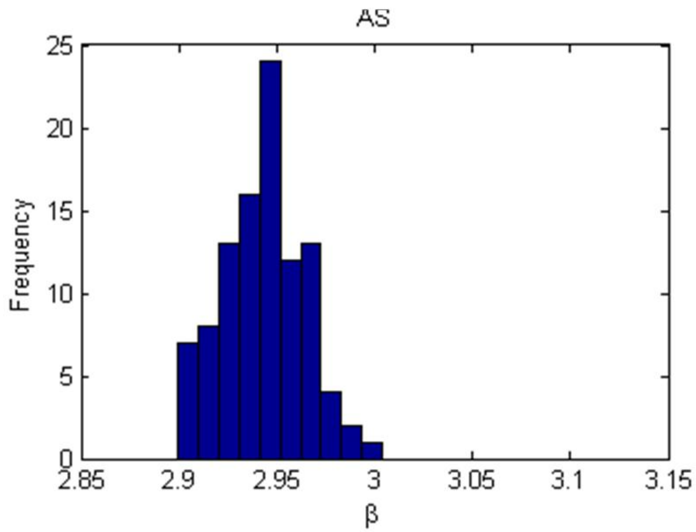
[2.79517, 3.12404]

100 run	SS	AS
Min β	2.9156	2.8914
Max β	3.0883	2.9896
Mean β	3.0015	2.9431
Standard deviation β	0.0370	0.0205
Coeff. of variation β	1.23 %	0.7 %
Number of samples	18,000	18,000

Single run with MC:

β	2.9972
CoV MC	1.92 %
Number of sim.	65,000





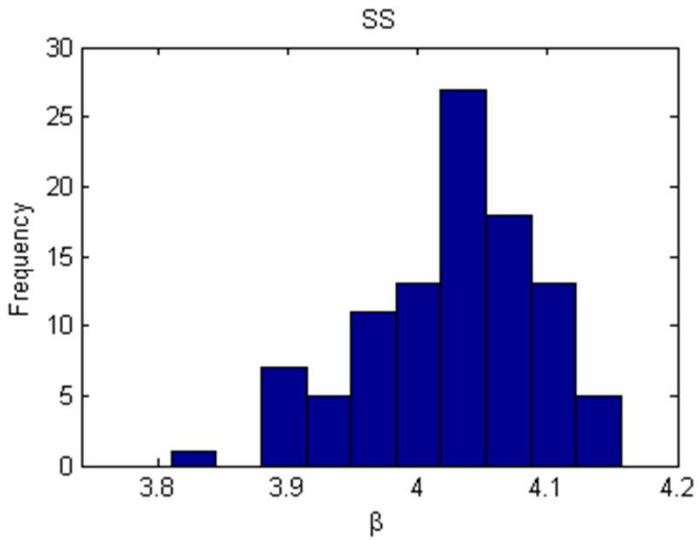
2D Benchmark: RBDO Comparison

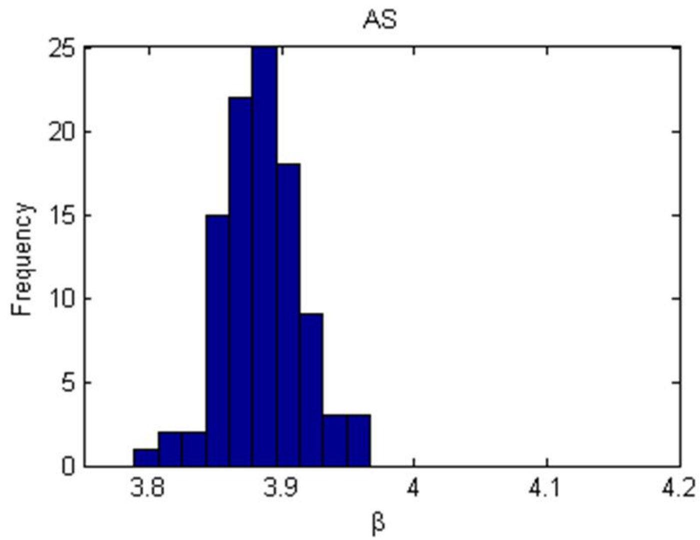
[2.76199, 2.97517]

100 run	SS	AS
Min β	3.8326	3.8024
Max β	4.1804	3.9598
Mean β	4.0438	3.8832
Standard deviation β	0.0632	0.0317
Coeff. of variation β	1.56 %	0.82 %
Number of samples	30,000	30,000

Single run with MC:

β	3.9818
CoV MC	3.82 %
Number of sim.	20,000,000





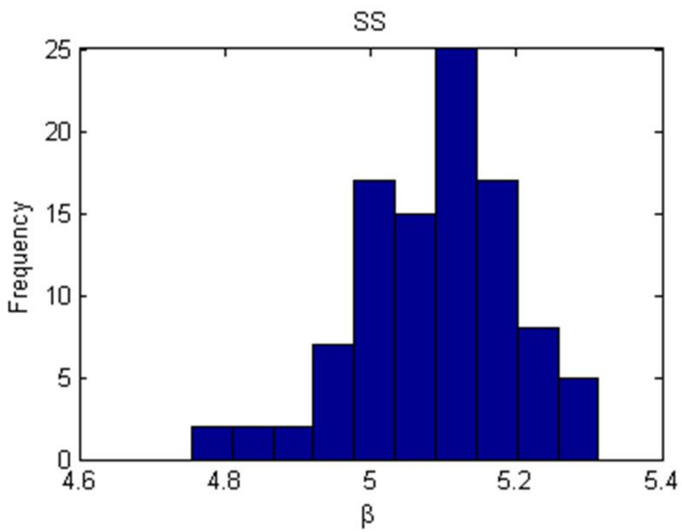
2D Benchmark: RBDO Comparison

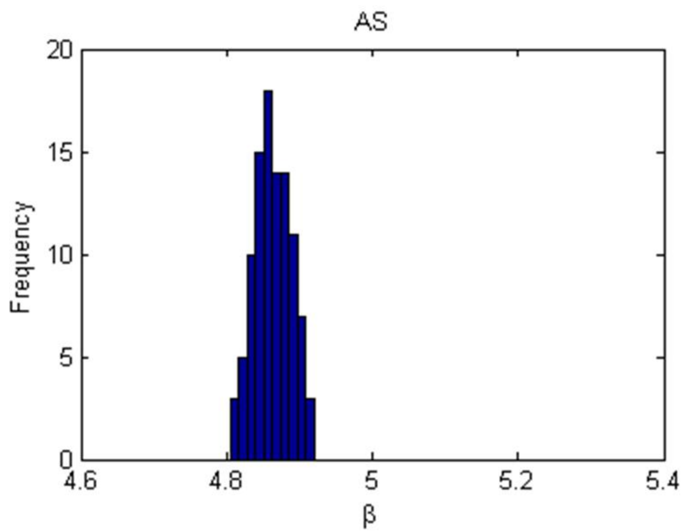
[2.74135, 2.78964]

100 run	SS	AS
Min β	4.7545	4.8058
Max β	5.3128	4.9297
Mean β	5.0902	4.8606
Standard deviation β	0.1054	0.0273
Coeff. of variation β	2.07 %	0.56 %
Number of samples	42,000	43,000

Single run with MC:

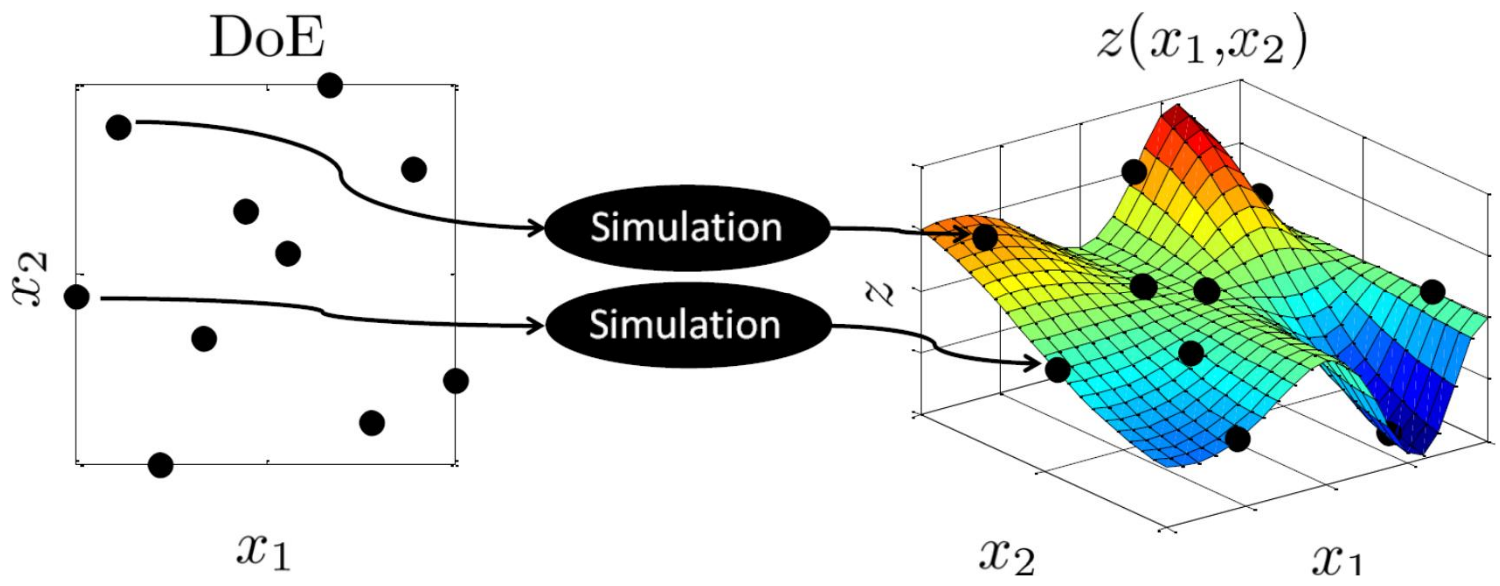
β	4.9905
CoV MC	13.62 %
Number of sim.	179,000,000





META-MODELS

- model of original model with same behaviour but easier (faster) to evaluate
- Original model still necessary to evaluate few times
- Choosing points where to enumerate original model - Design of Experiments (DoE)



STARTING DESIGN OF EXPERIMENTS

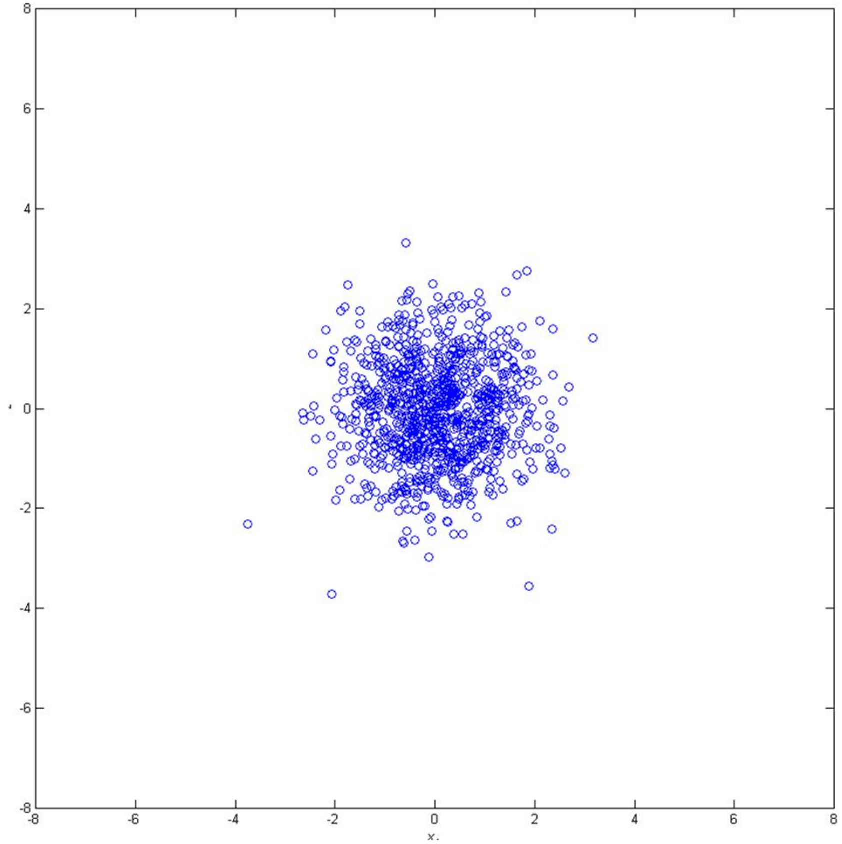
- Sampling from prescribed distributions

✓Known methodology

✘Sampling around mean

✘May miss failure region

✘Problems with adaptive sampling



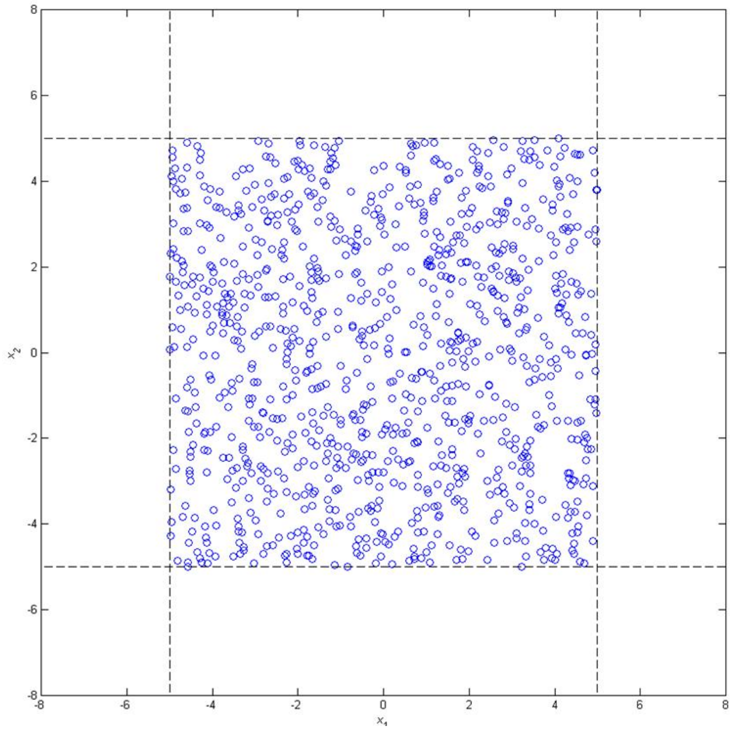
Starting Design of Experiments

- Sampling from hypercube

✓Known methodology

✓Fast and simple

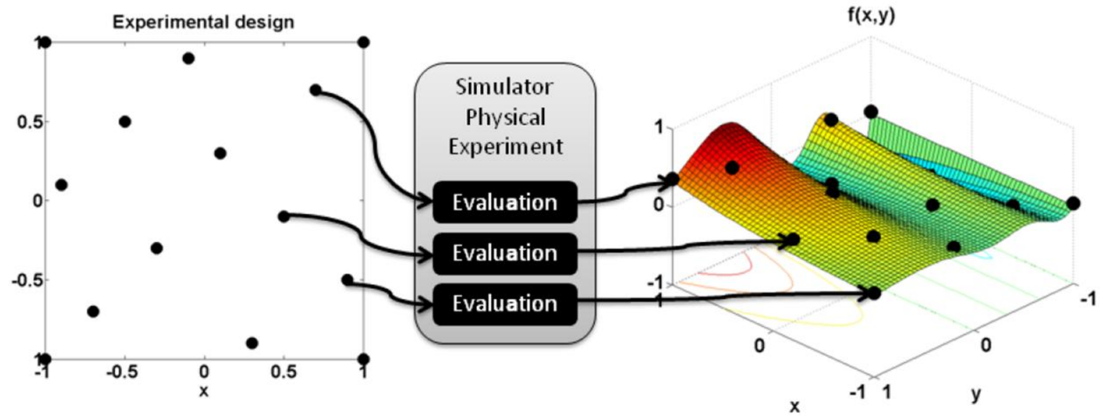
✓Enables adaptive sampling



✘Omits solutions outside bounds!

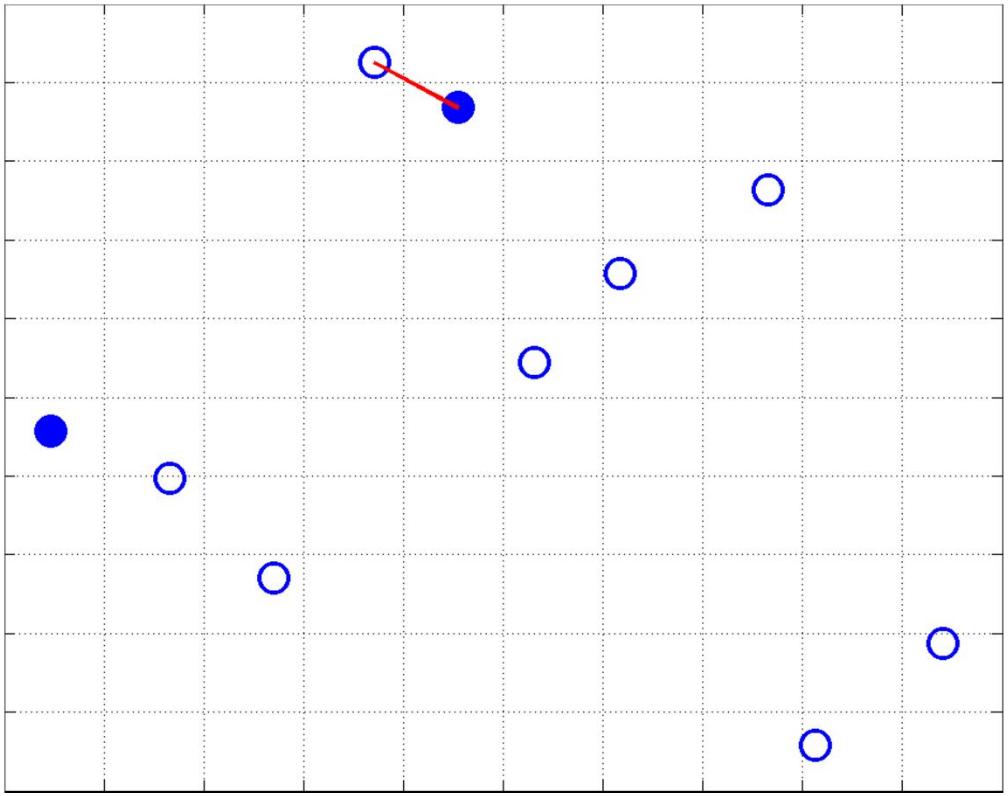
Hypercube Design of experiments

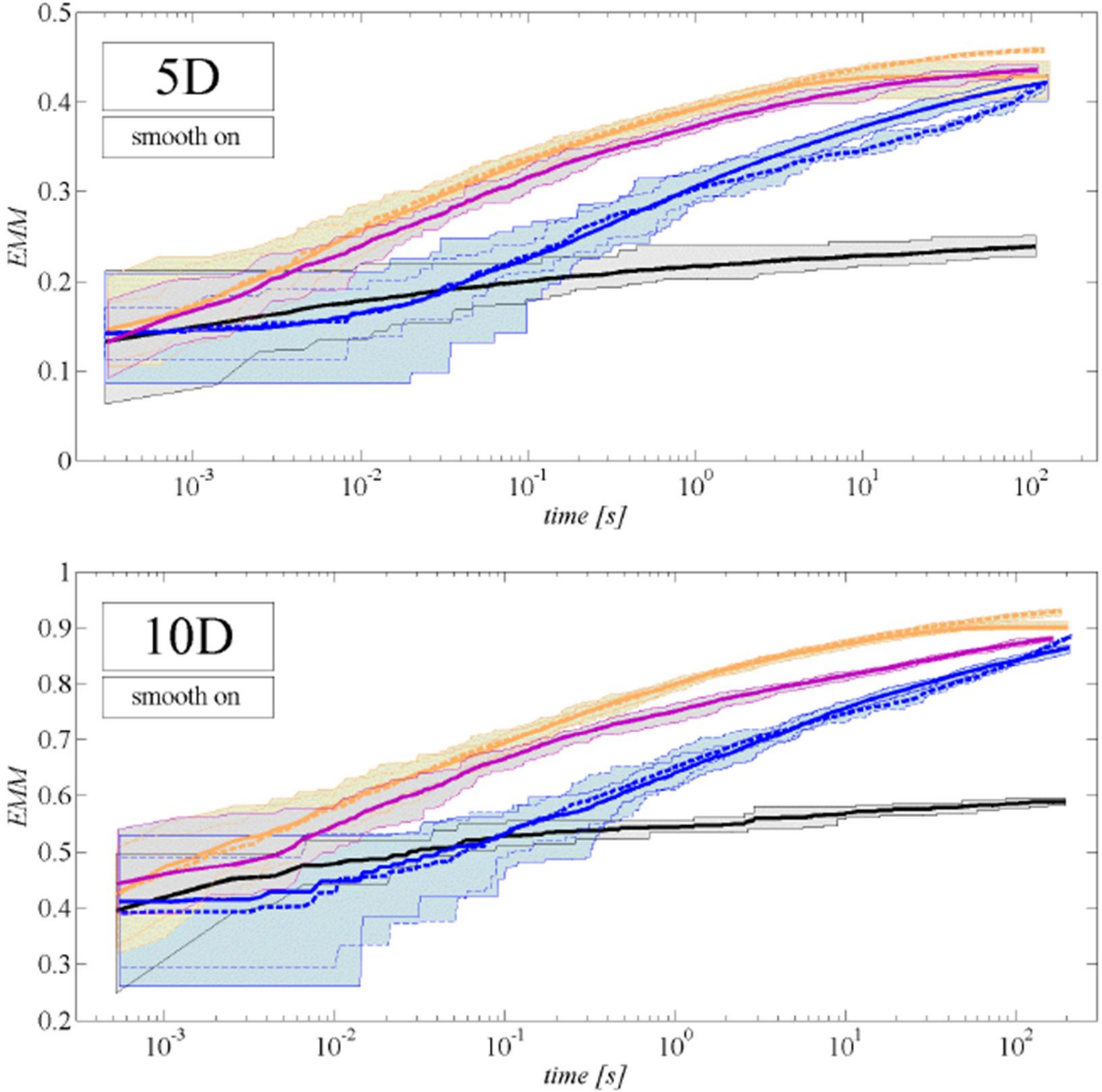
- essential part of surrogate modeling and simulations



- Implemented:
  - Pure random
  - Halton and Sobol sequences
  - LHS
    - Standard Matlab
    - Optimized w.r.t. EMM and other criterions

Optimized LHS – heuristic procedure plus Simulated annealing




5D
smooth on
EMM
time [s]
0
0.1
0.2
0.3
0.4
0.5
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10D
smooth on
EMM
time [s]
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}

HYPERCUBE DESIGN OF EXPERIMENTS

- different metrics for comparison of quality implemented during project
  - AE, CN, corr, KRCC, PMCC, SRCC, ML2, EMM, miniMax
  - correlations also used for Sensitivity Analysis

DIFFERENCE BETWEEN EMM AND MM

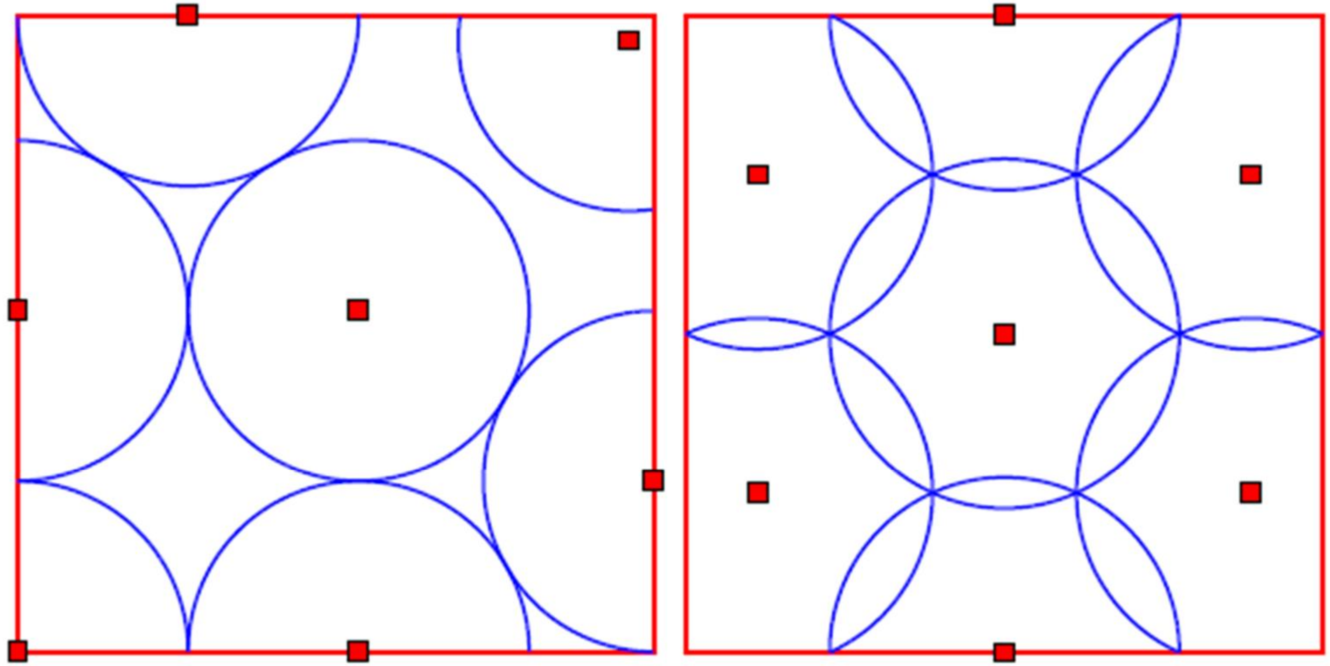


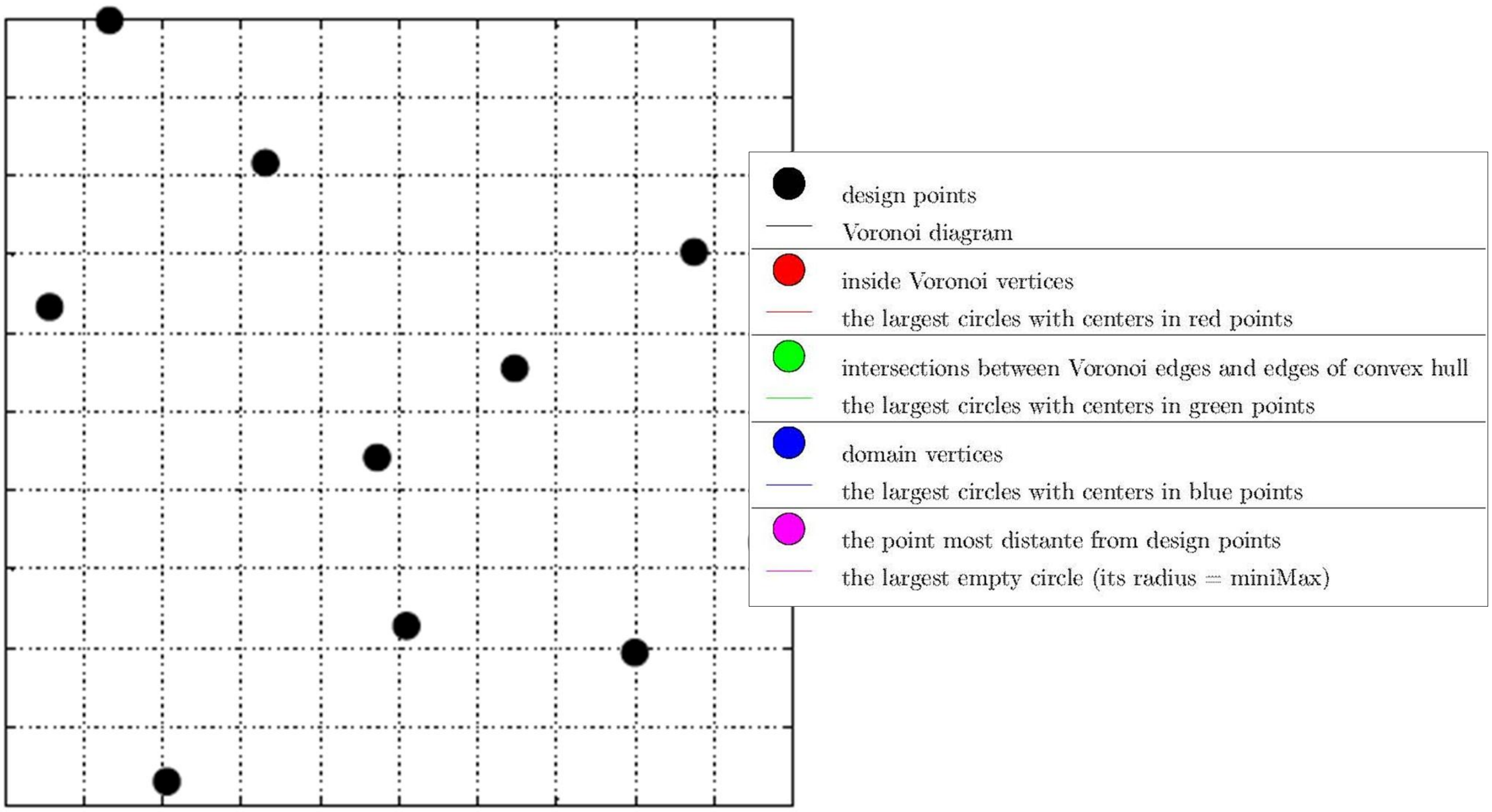
Fig. 1 Maximin (*left*, see http://www.packomania.com/ and minimax (*right*, see Johnson et al. 1990) distance designs for $n = 7$ points in $[0, 1]^2$. The *circles* have radius $\phi_{Mm}(\xi)/2$ on the *left panel* and radius $\phi_{mM}(\xi)$ on the *right one*

[Pronzato, 2012]

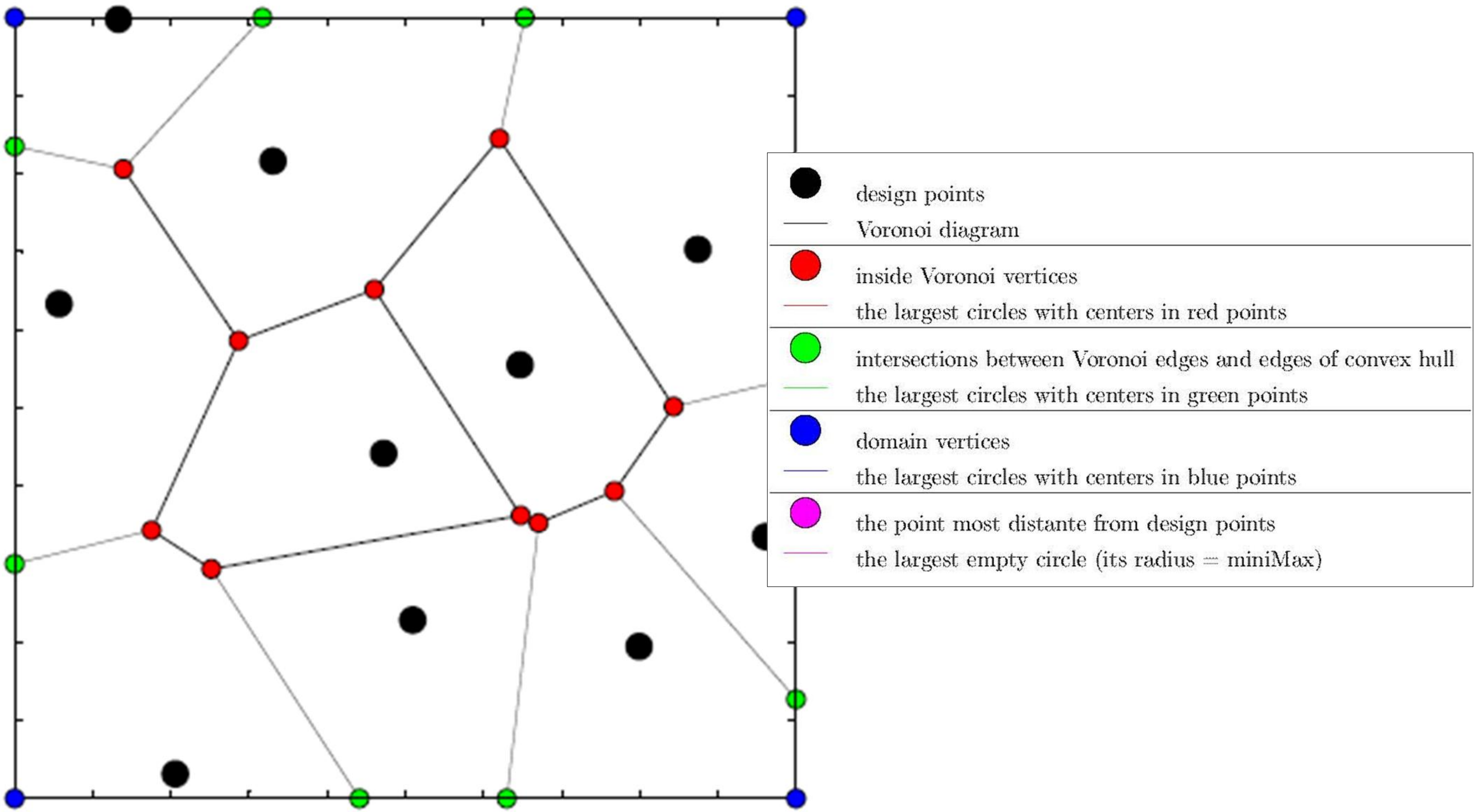
MINIMAX

- Can be found as “the largest empty circle problem”
- Centers of circles (spheres) coincides with the vertices of the Voronoi diagrams

miniMax I


design points
Voronoi diagram
inside Voronoi vertices
the largest circles with centers in red points
intersections between Voronoi edges and edges of convex hull
the largest circles with centers in green points
domain vertices
the largest circles with centers in blue points
the point most distante from design points
the largest empty circle (its radius = miniMax)

miniMax I



design points
Voronoi diagram
inside Voronoi vertices
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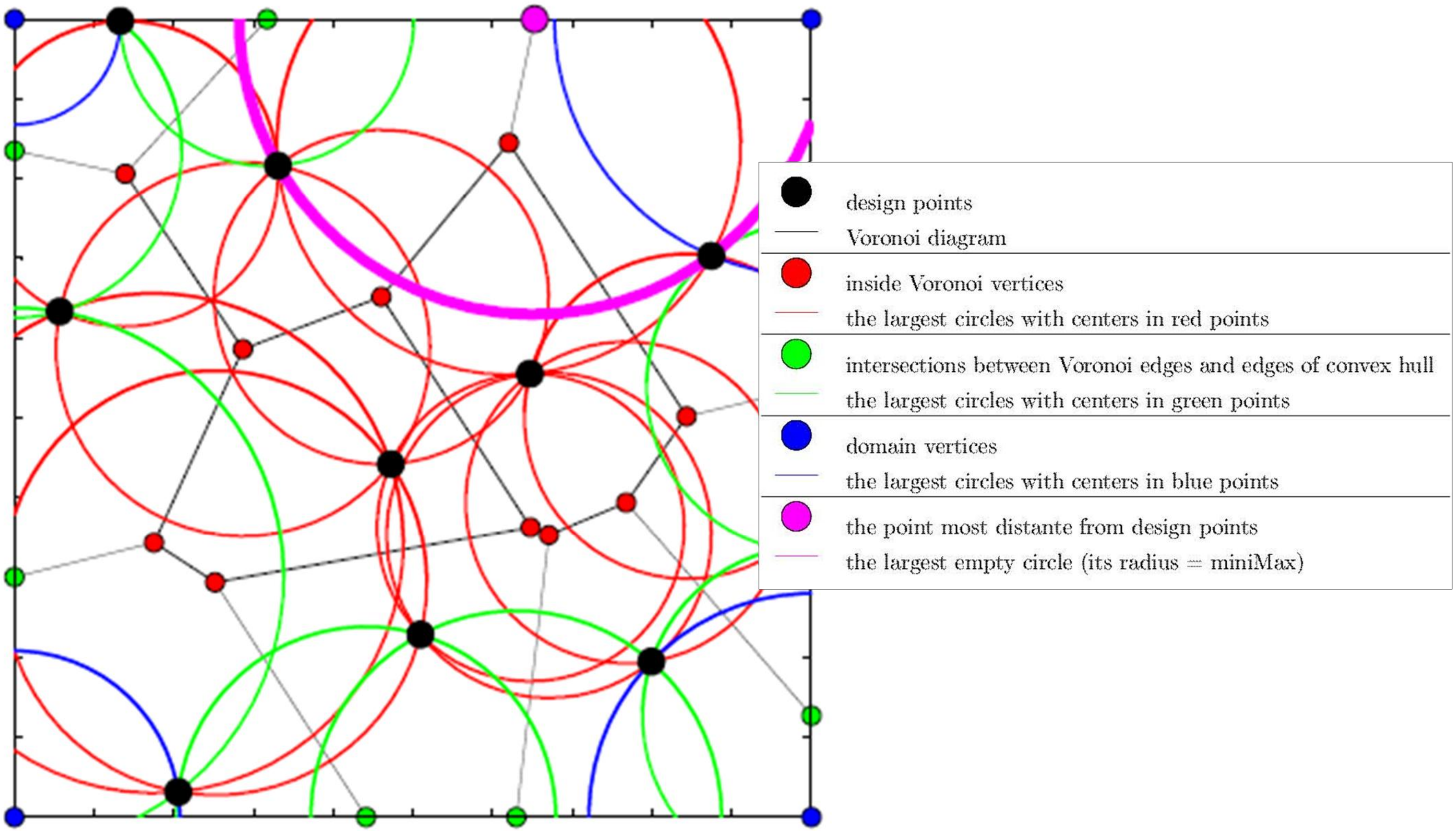
the largest circles with centers in blue points
the point most distante from design points
the largest empty circle (its radius = miniMax)

miniMax I


design points
Voronoi diagram
inside Voronoi vertices
the largest circles with centers in red points
intersections between Voronoi edges and edges of convex hull
the largest circles with centers in green points
domain vertices
the largest circles with centers in blue points
the point most distante from design points
the largest empty circle (its radius = miniMax)

COMPUTATIONAL ASPECTS

- Memory complexity: $O(n^{\lceil d/2 \rceil})$

LHS 100 p. Design

Dimension	Time [s]	Memory [kB]
2D	0.082	684
3D	0.085	1576
4D	0.393	9540
5D	3.885	15796
6D	150.924	71916
7D	6297.79	454236
8D	> 6 d. 18 h.	> 8 GB

COMPUTATIONAL ASPECTS

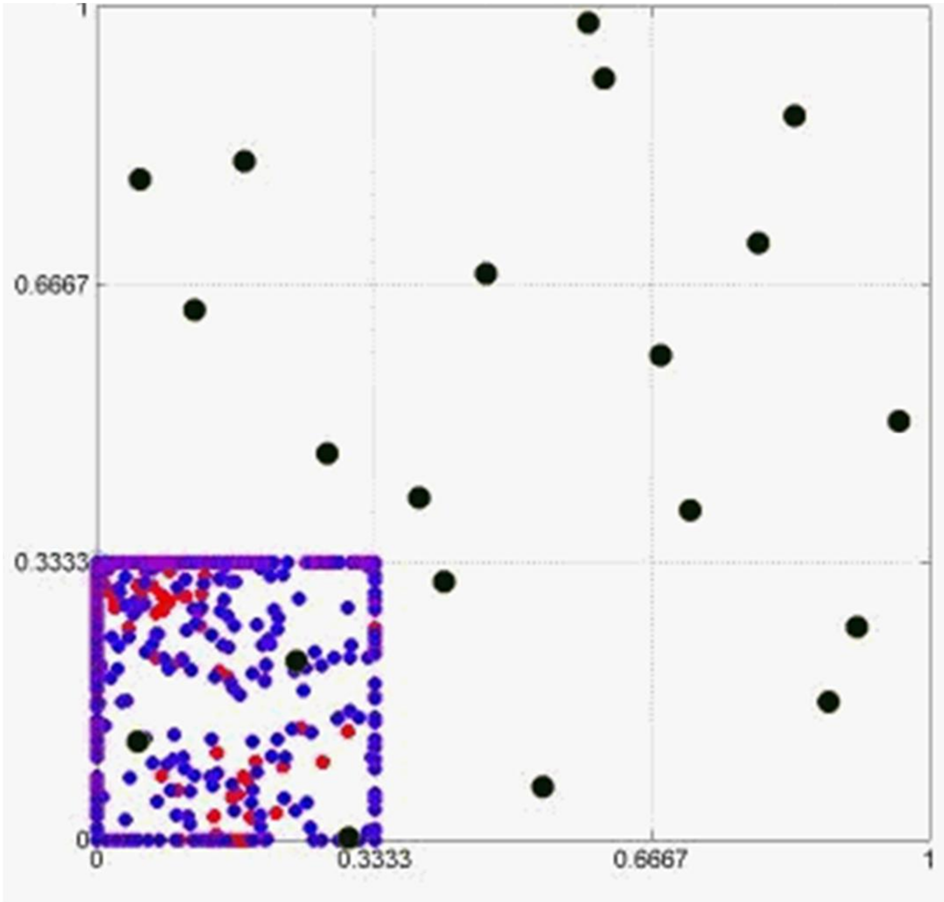
Prediction by exponencial fc.

Dimension	Time	Memory [GB]
9D	117 days	11
10D	12,9 yrs	56
11D	521 yrs	286
12D	20977 yrs	1451
...		
35D	??	??

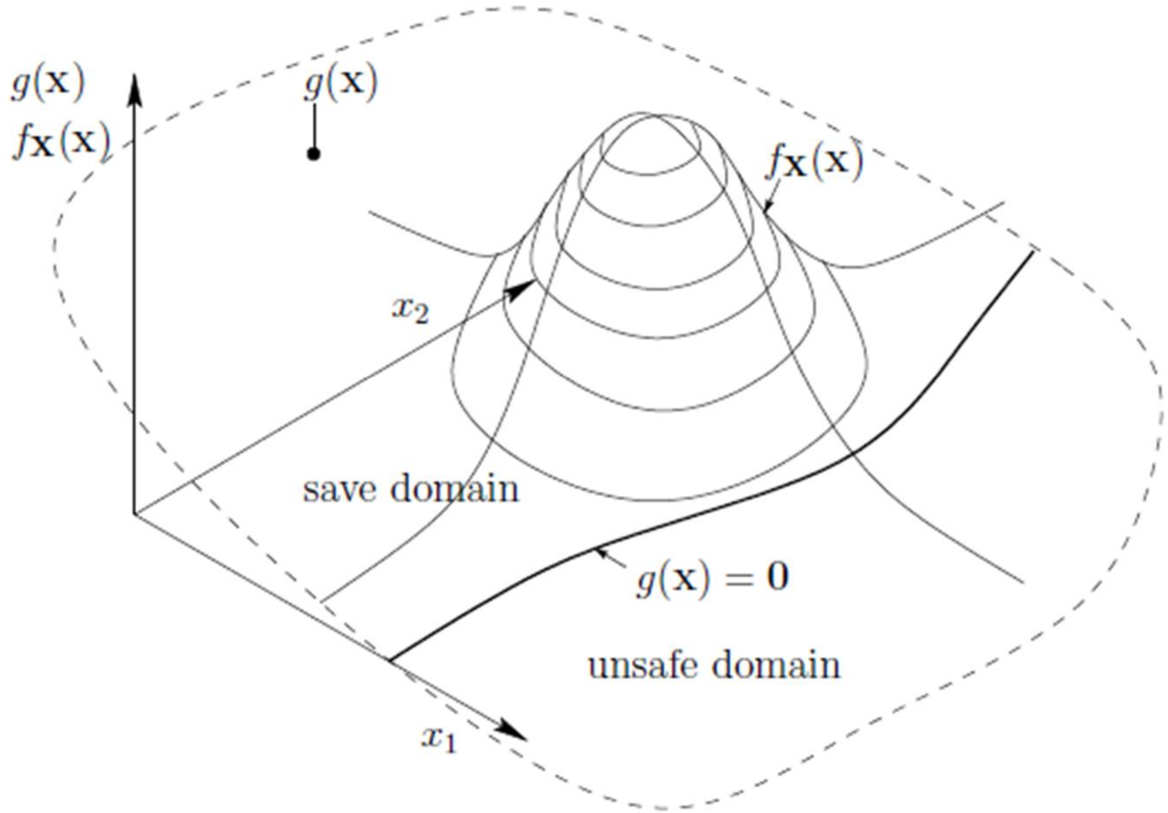
APPROXIMATE MINIMAX

- uses parallel Evolution Strategy
- efficient in terms of execution time and necessary memory in comparison with the Voronoi diagram approach

APPROXIMATE MINIMAX



ADAPTIVE SAMPLING AROUND **LIMIT STATE FUNCTION**



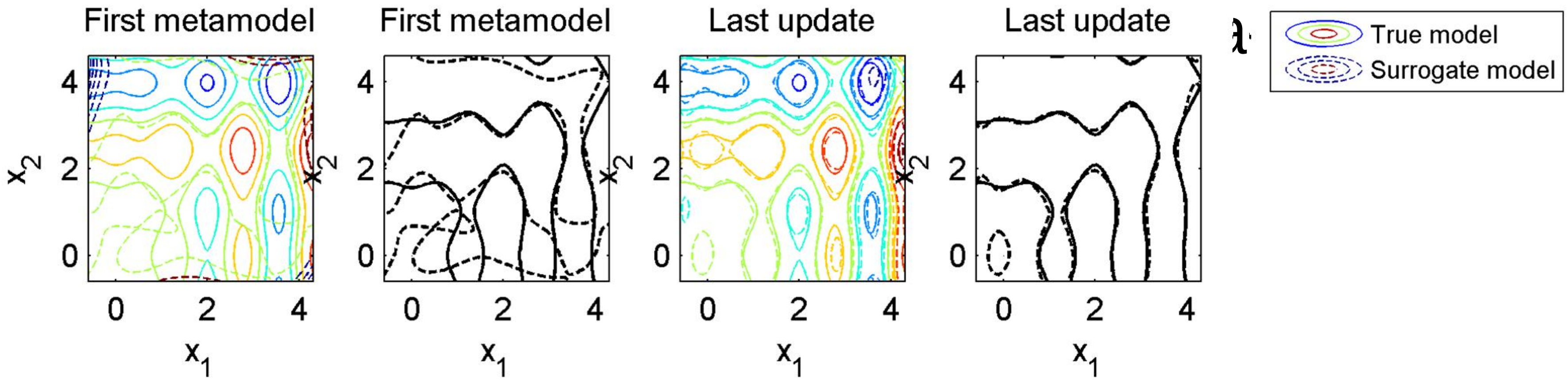
[Roos, 2006]

SURROGATE MODEL

- Appropriate number of sampling points is needed
- Adaptive updating procedure
  - Multi-objective optimization problem
  - Maximization of the nearest distance of the added point from already sampled points (like miniMax metric)
  - To be as close as possible to the approximate limit state surface

ADAPTIVE MULTI-OBJECTIVE OPTIMIZATION UPDATING PROCEDURE

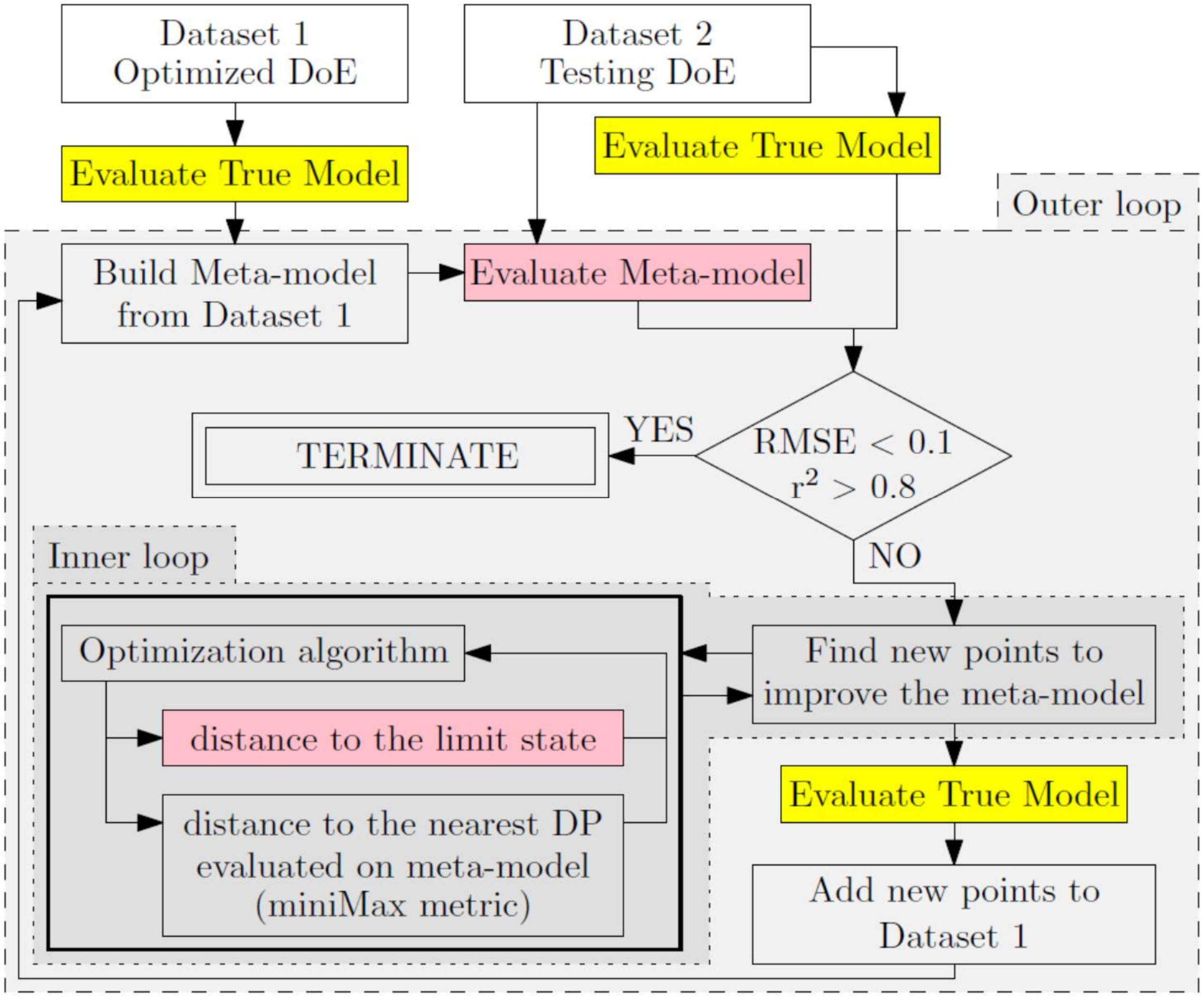
- One global meta-model built and updated separately from the optimization procedure



- For all meta-model types
- Several finite number of points for update in one step
- Parallelizable

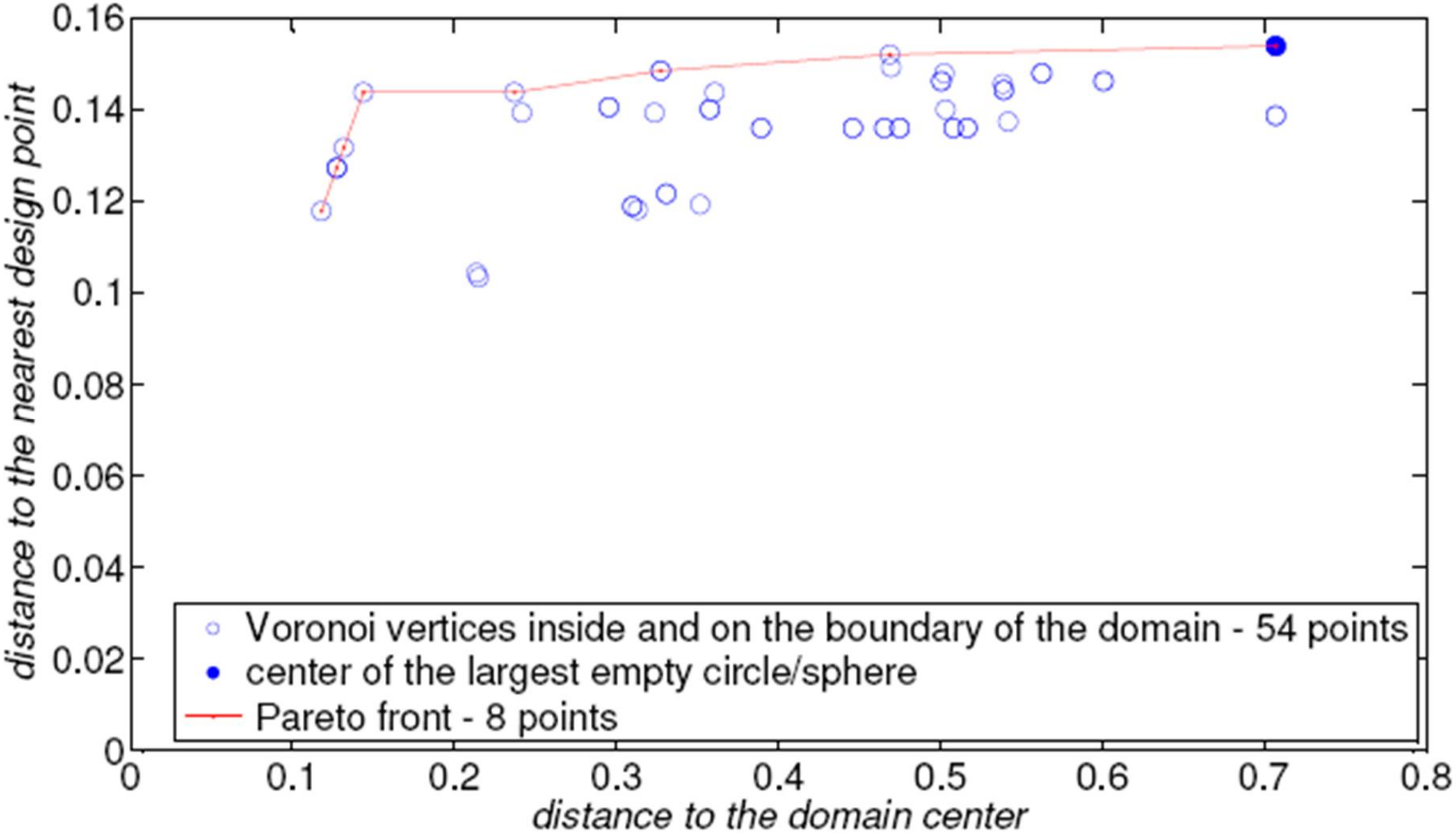
- Limit state surface has to be precise in whole domain
- ⇒ Plenty of points have to be added to DoE
- ⇒ A large system of equations

Adaptive Multi-Objective Optimization Updating Procedure



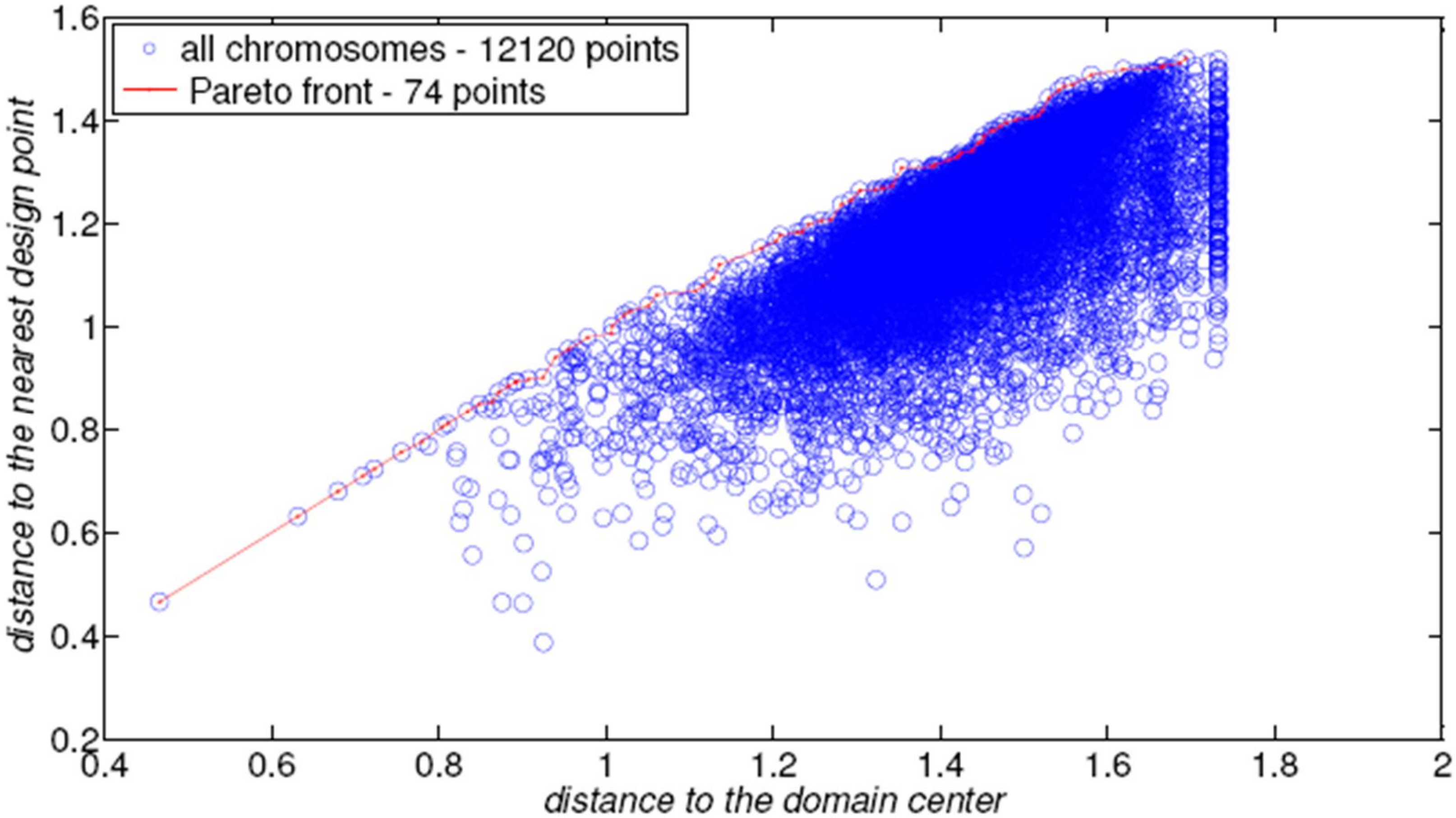
Multi-objective adaptive sampling

2D, 27 points



MULTI-OBJECTIVE ADAPTIVE SAMPLING

12D, 65 points



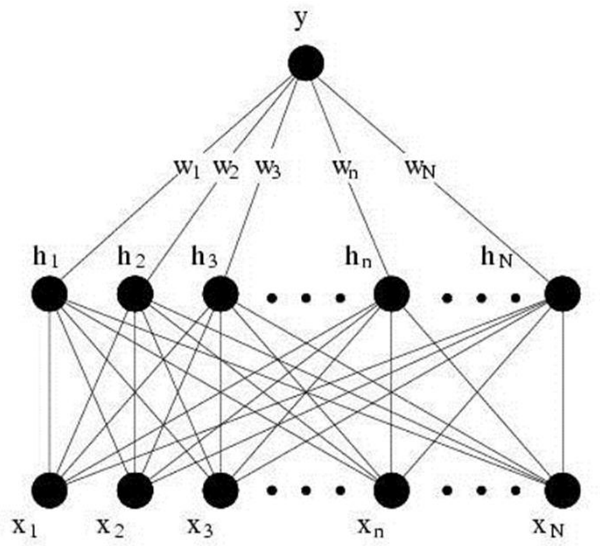
Implemented Meta-Models

- RBFN from Matlab
  - Neural Network based
- CTU implementation of RBFN
  - with different polynomial regression parts
- Kriging
  - DACE toolbox in Matlab
  - with different polynomial regression parts
  - with regression part found by Genetic Programming

RBFN (Radial-Basis Function Network)

- Weights $\boldsymbol{w}_i$ computed from equality of approximation and original function in training points ... leads to **a system of linear equations**

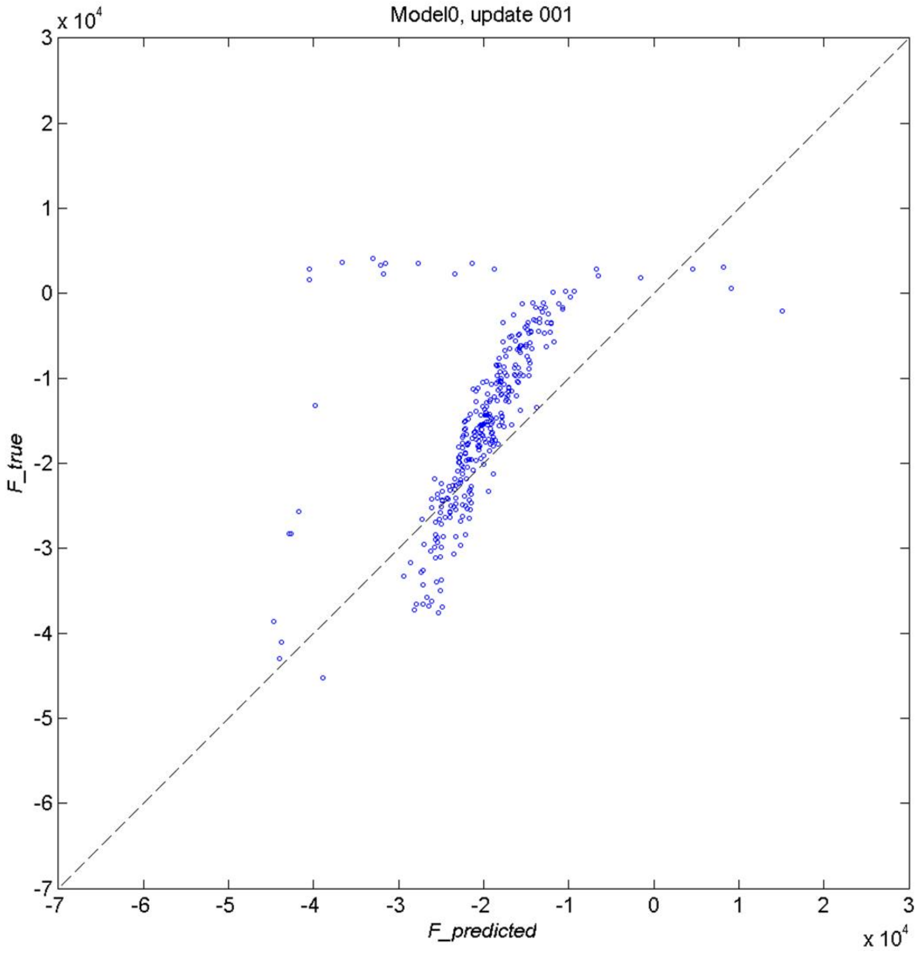
$$\widehat{y}(\mathbf{x}) = \sum_{i=1}^{N} w_i h_i(\mathbf{x})$$



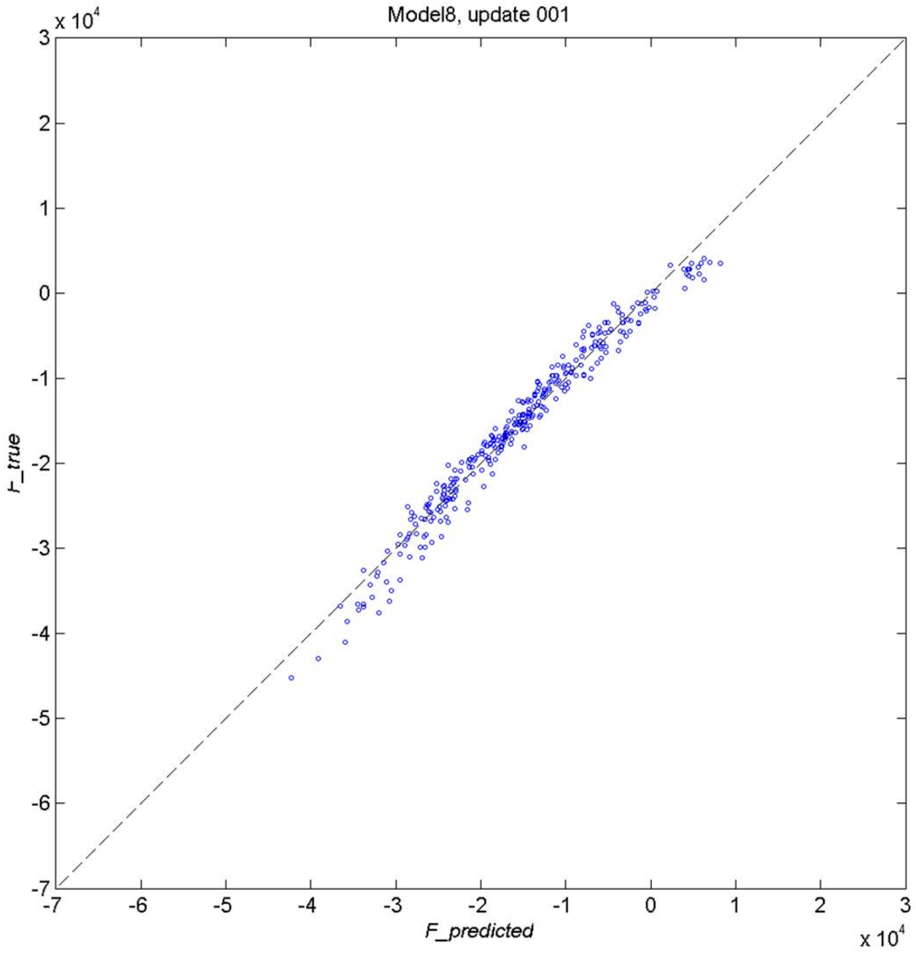
Basis function : $$h_i(\mathbf{x}) = e^{-\|\mathbf{x}-\mathbf{x}_i\|^2/r}$$

QUALITY OF A METAMODEL

RBFN (Matlab)

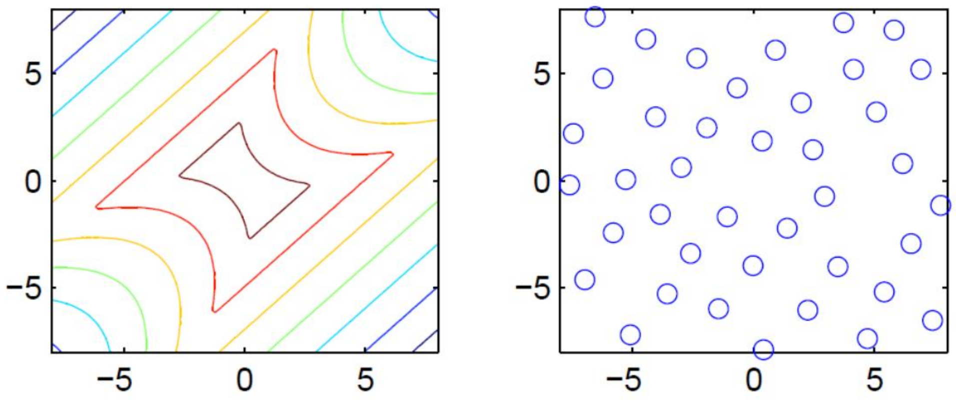


Kriging

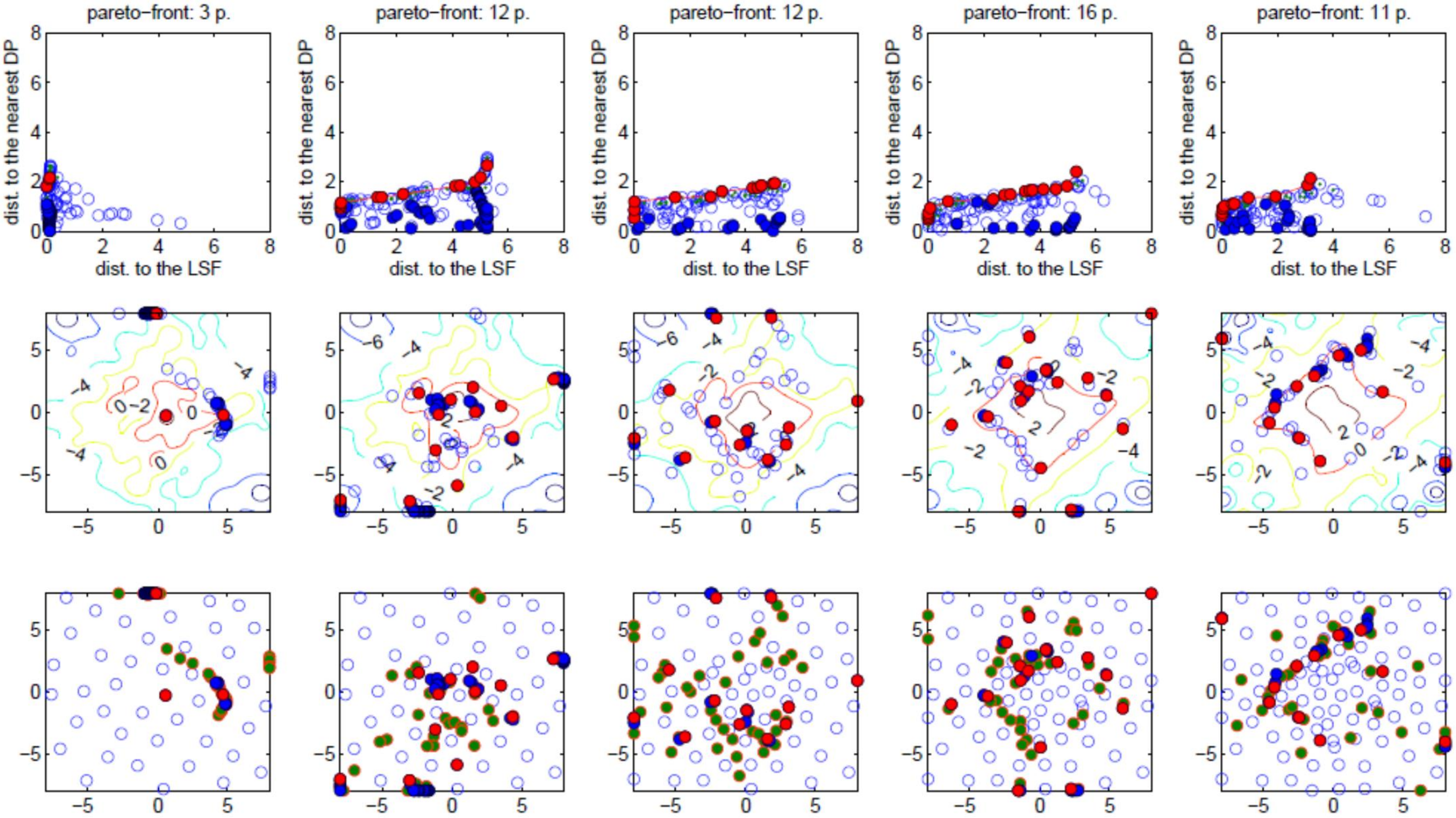


BENCHMARK

$$F(x) = min \begin{pmatrix} 3 + \frac{(x_1 - x_2)^2}{10} - \frac{(x_1 + x_2)}{\sqrt{2}} \\ 3 + \frac{(x_1 - x_2)^2}{10} + \frac{(x_1 + x_2)}{\sqrt{2}} \\ x_1 - x_2 + \frac{7}{\sqrt{2}} \\ x_2 - x_1 + \frac{7}{\sqrt{2}} \end{pmatrix}, x \in [-8,8]^2$$

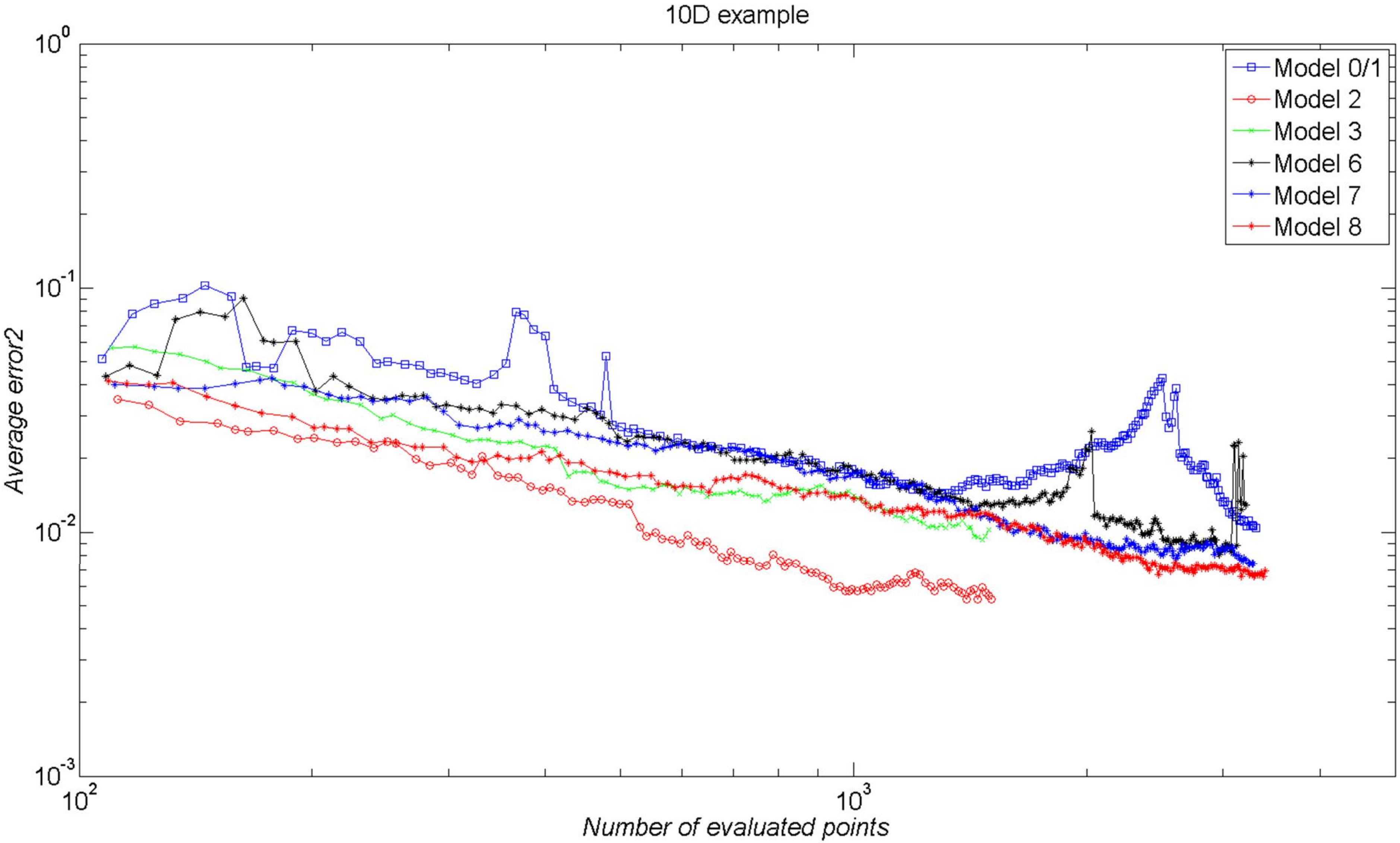


Contours of the example (left) and starting DoE (right). Note that the red contour is for F(x) = 0.

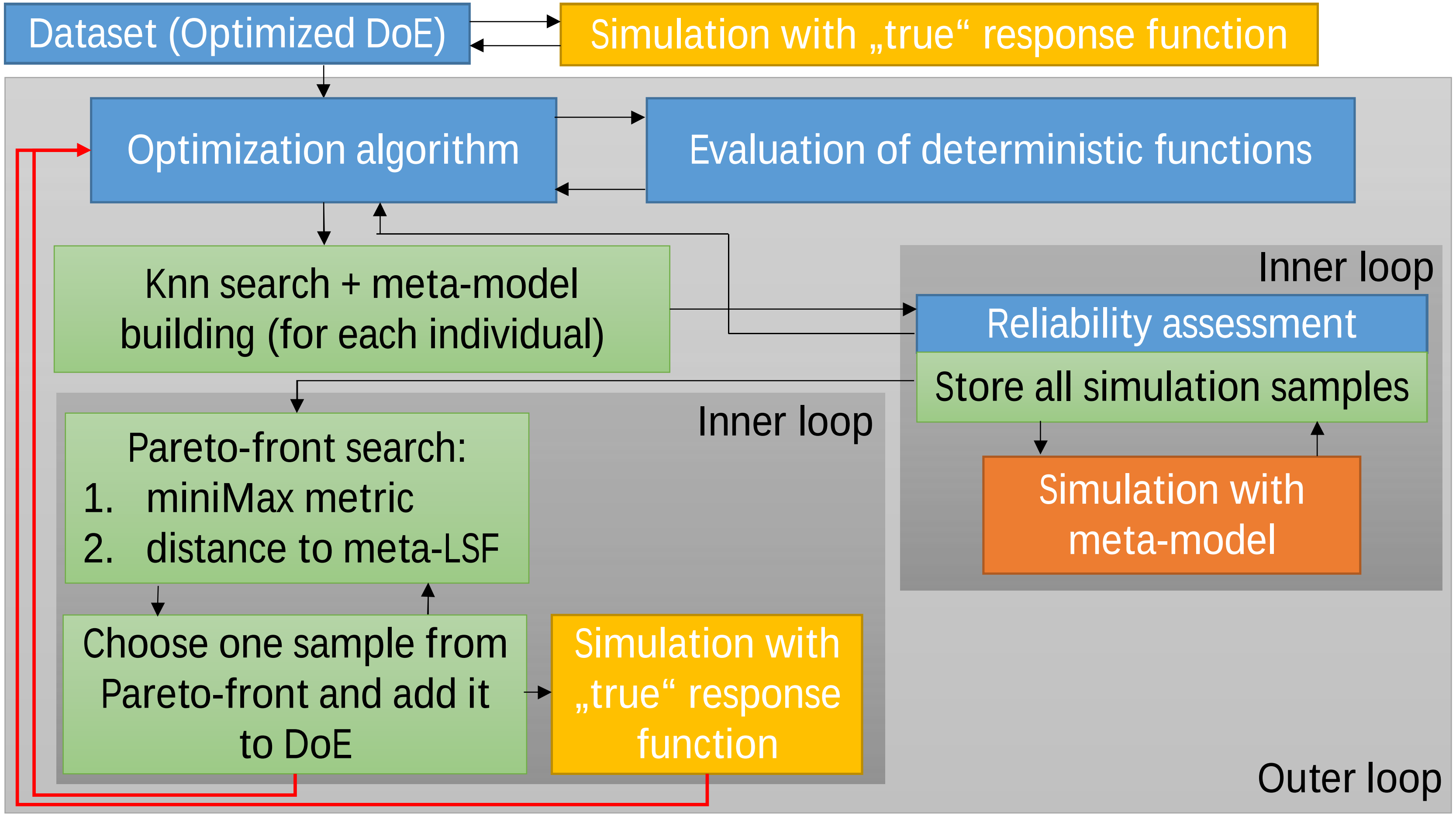


Pareto front (top), contours of the problem with DoEs (middle) and DoEs' points (bottom).
Key: Red – added and computed solutions, Blue – points that were too close to other Pareto front points, Green – the remaining points of population and Blue empty points – the original DoE.

QUALITY OF UPDATING PROCEDURE



USAGE OF LOCAL META-MODEL



2D Benchmark: 2 Design v., 2 Stochastic v.

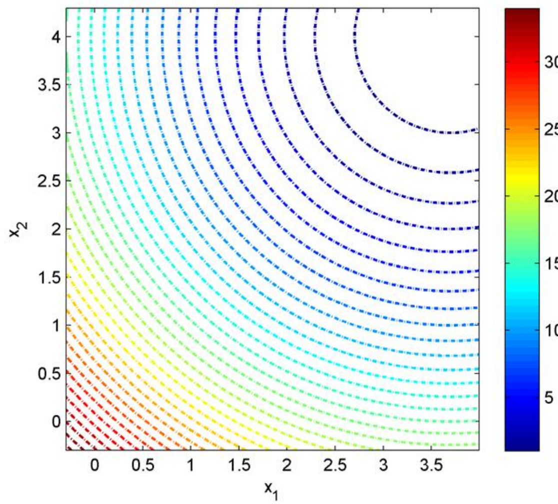
$\min\ (\mu_{X1} - 3.7)^2 + (\mu_{X2} - 4)^2$

$\max \beta$

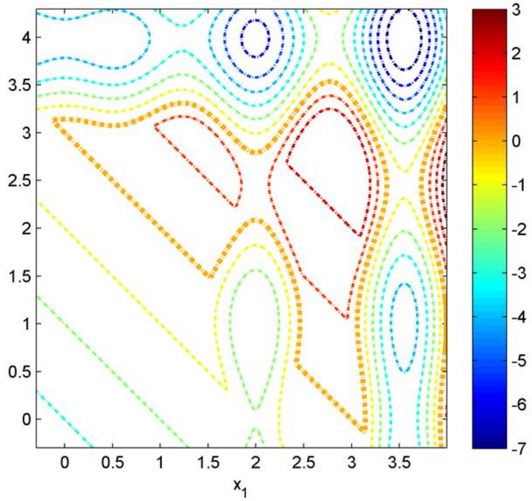
considering LSF $\quad \min \begin{pmatrix} -X_1 \sin(4X_1) - 1.1X_2 \sin(2X_2) \\ X_1 + X_2 - 3 \end{pmatrix}$

$0 \le \mu_{X1} \le 3.7, 0 \le \mu_{X2} \le 4$

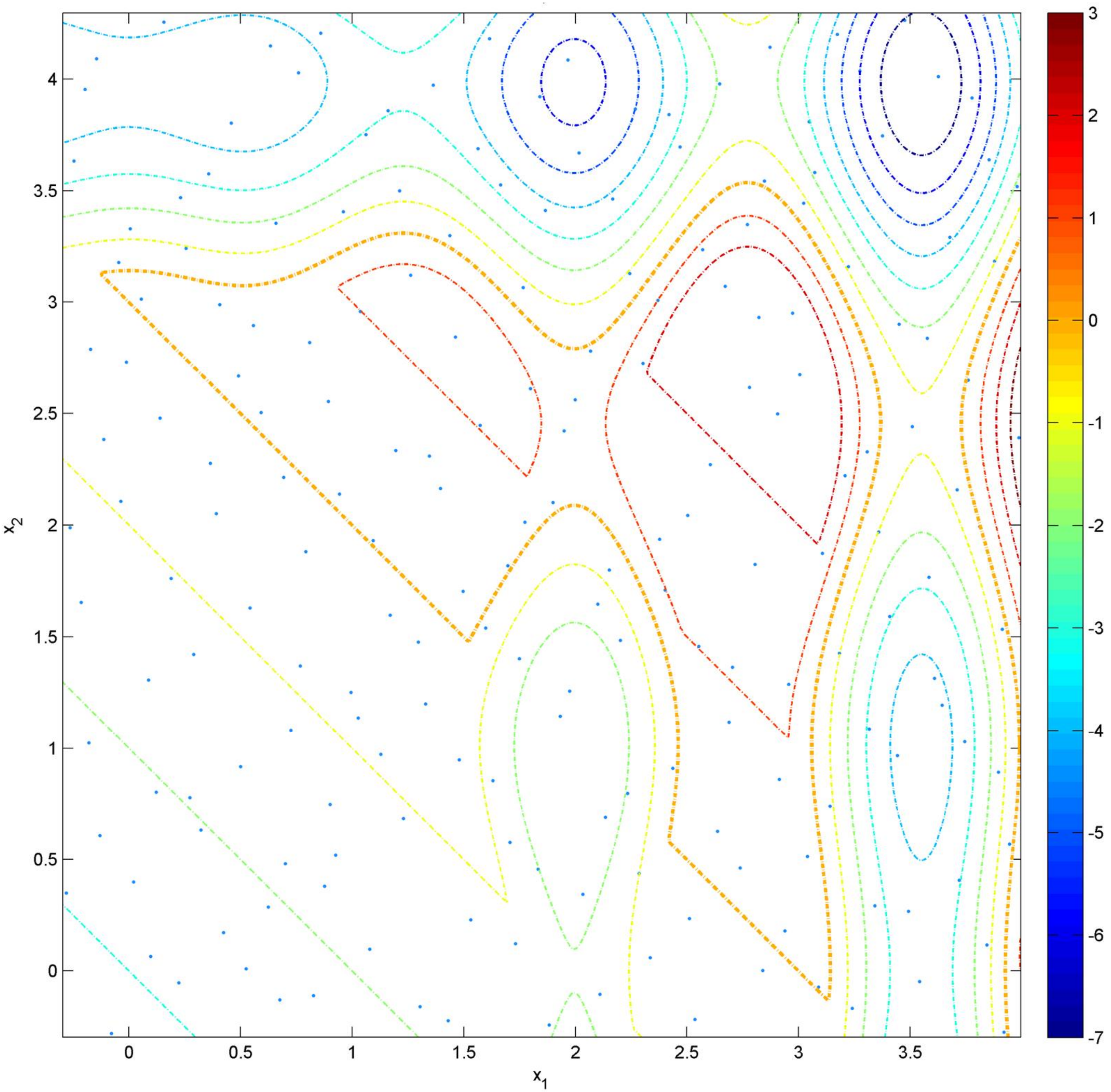
$(\mu_{X1} - 3.7)^2 + (\mu_{X2} - 4)^2$



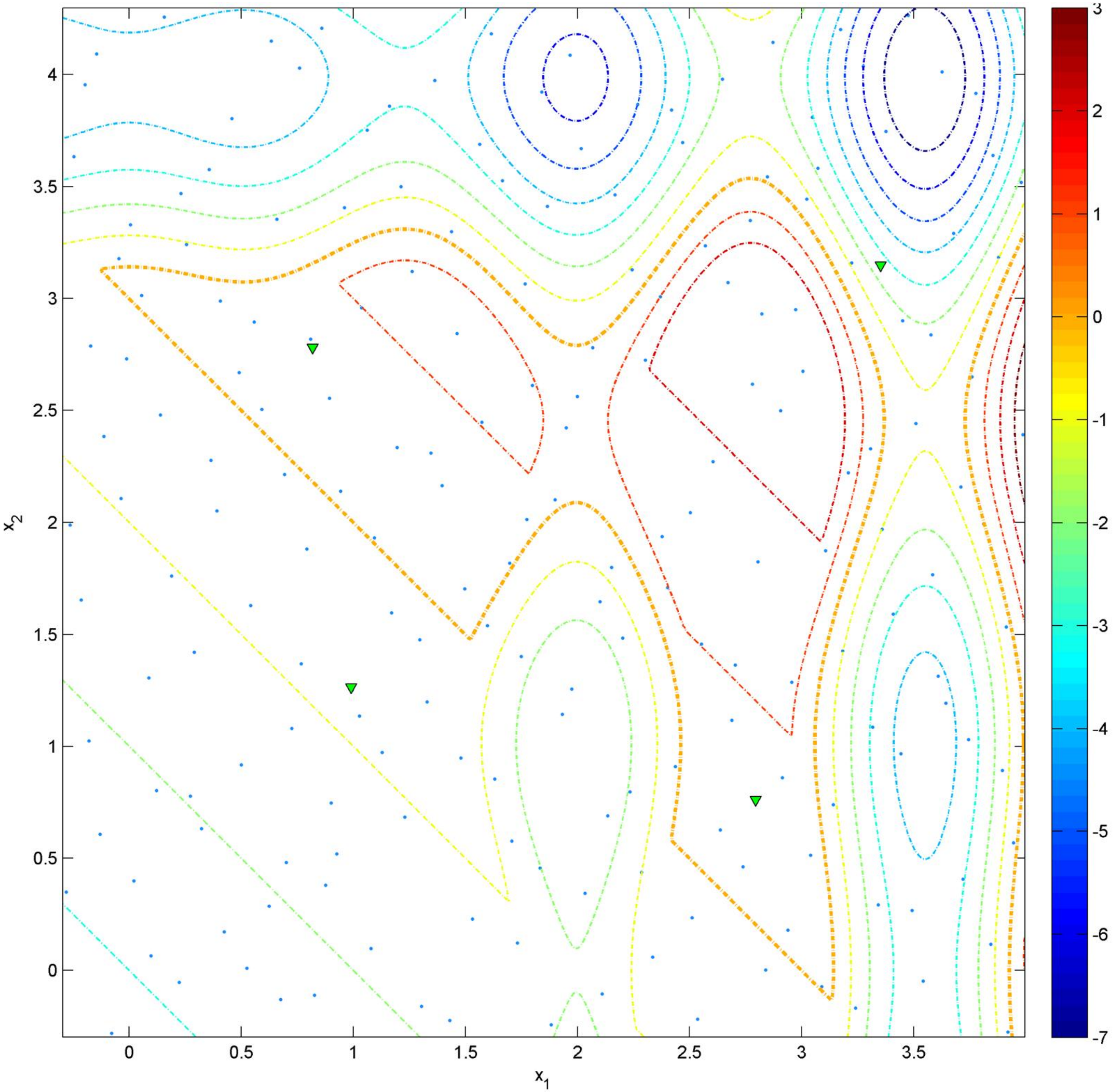
LSF



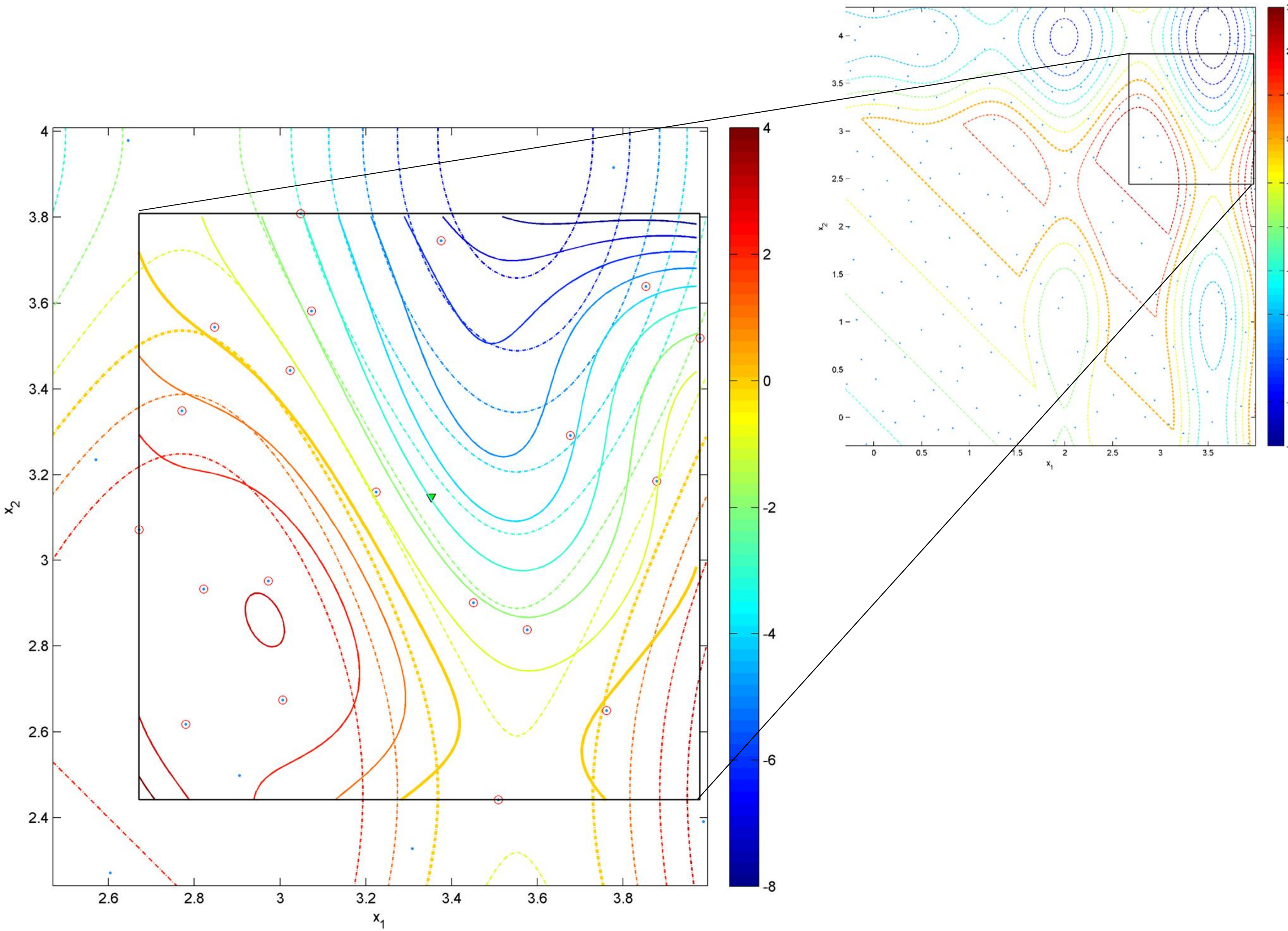
orange contour is for $g(\mathbf{X}) = 0$



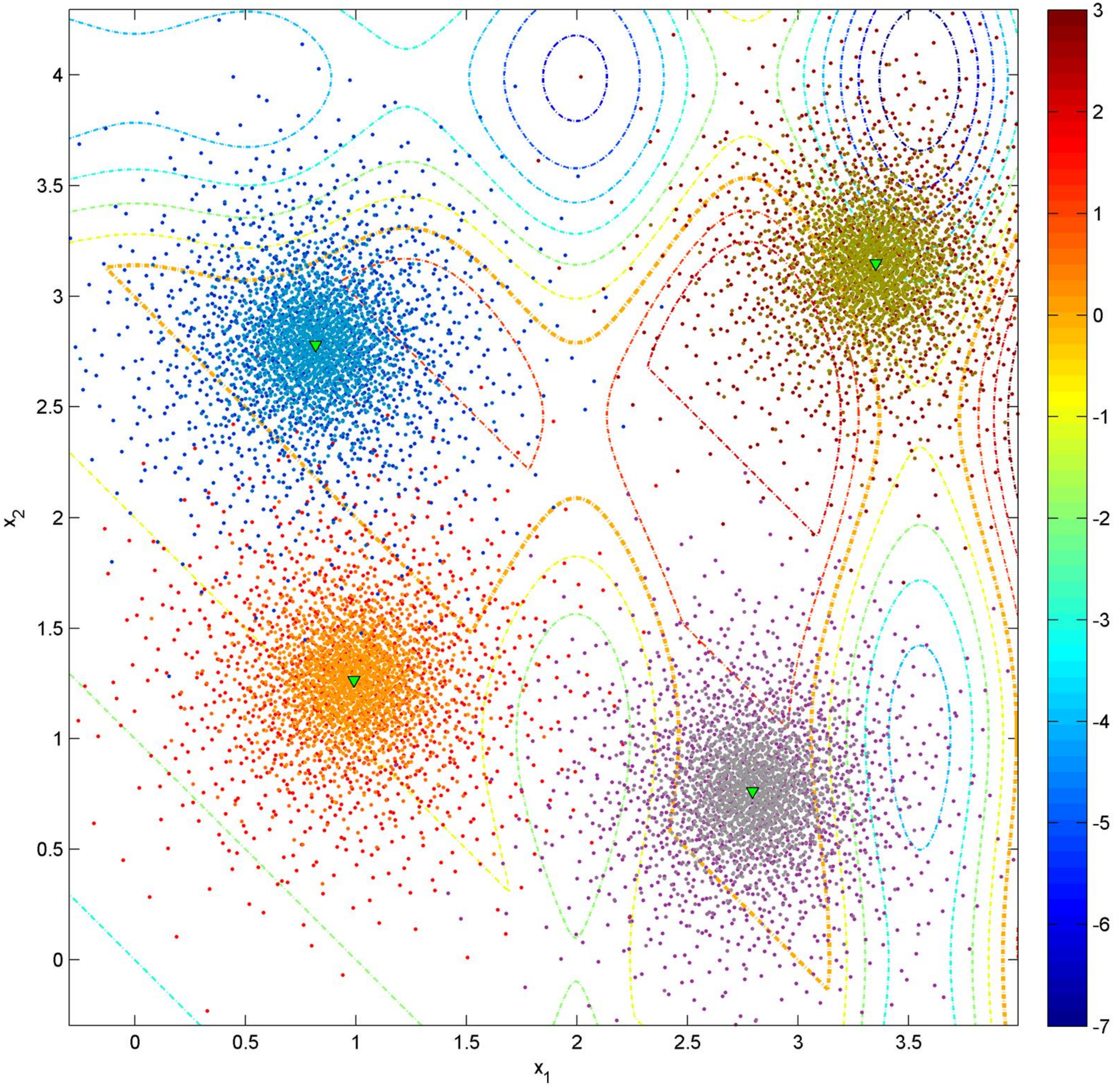
4
3.5
3
2.5
2
1.5
1
0.5
0
x_2
0
0.5
1
1.5
2
2.5
3
3.5
x_1
3
2
1
0
-1
-2
-3
-4
-5
-6
-7



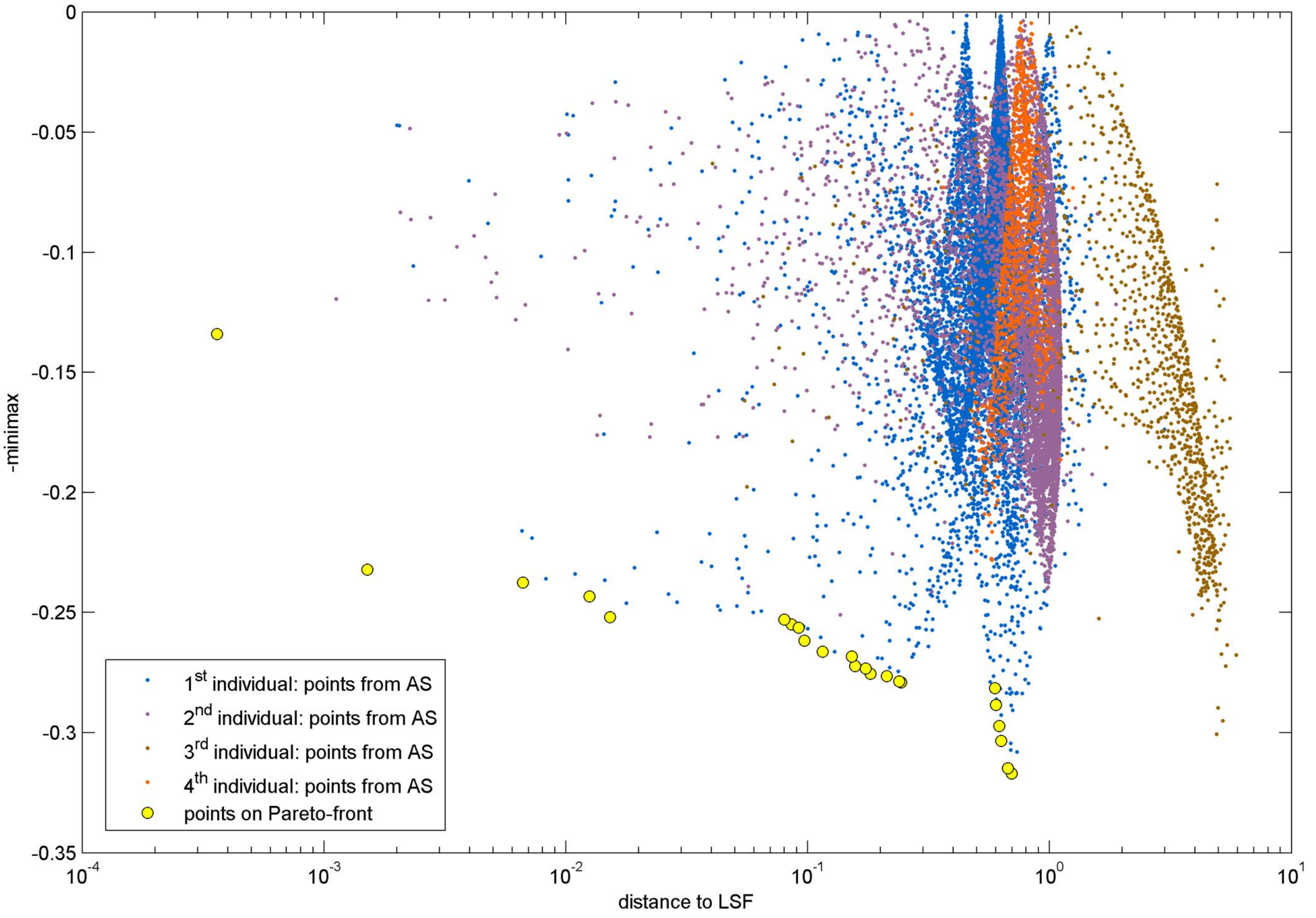
4
3.5
3
2.5
2
1.5
1
0.5
0
x_2
0
0.5
1
1.5
2
2.5
3
3.5
x_1
3
2
1
0
-1
-2
-3
-4
-5
-6
-7



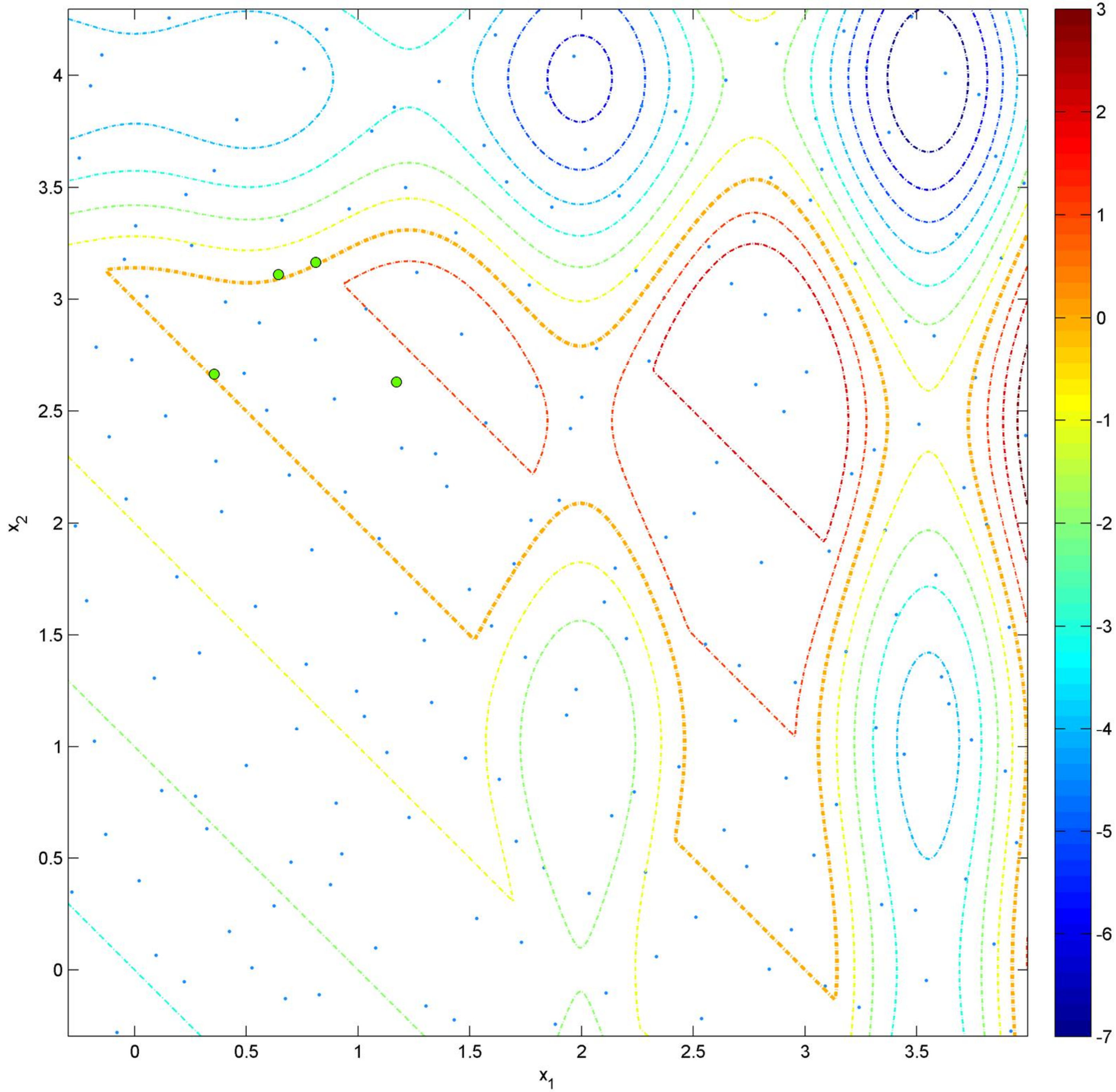
x1
x2
4
2
0
-2
-4
-6
-8
3
1
-1
-3
-5
-7



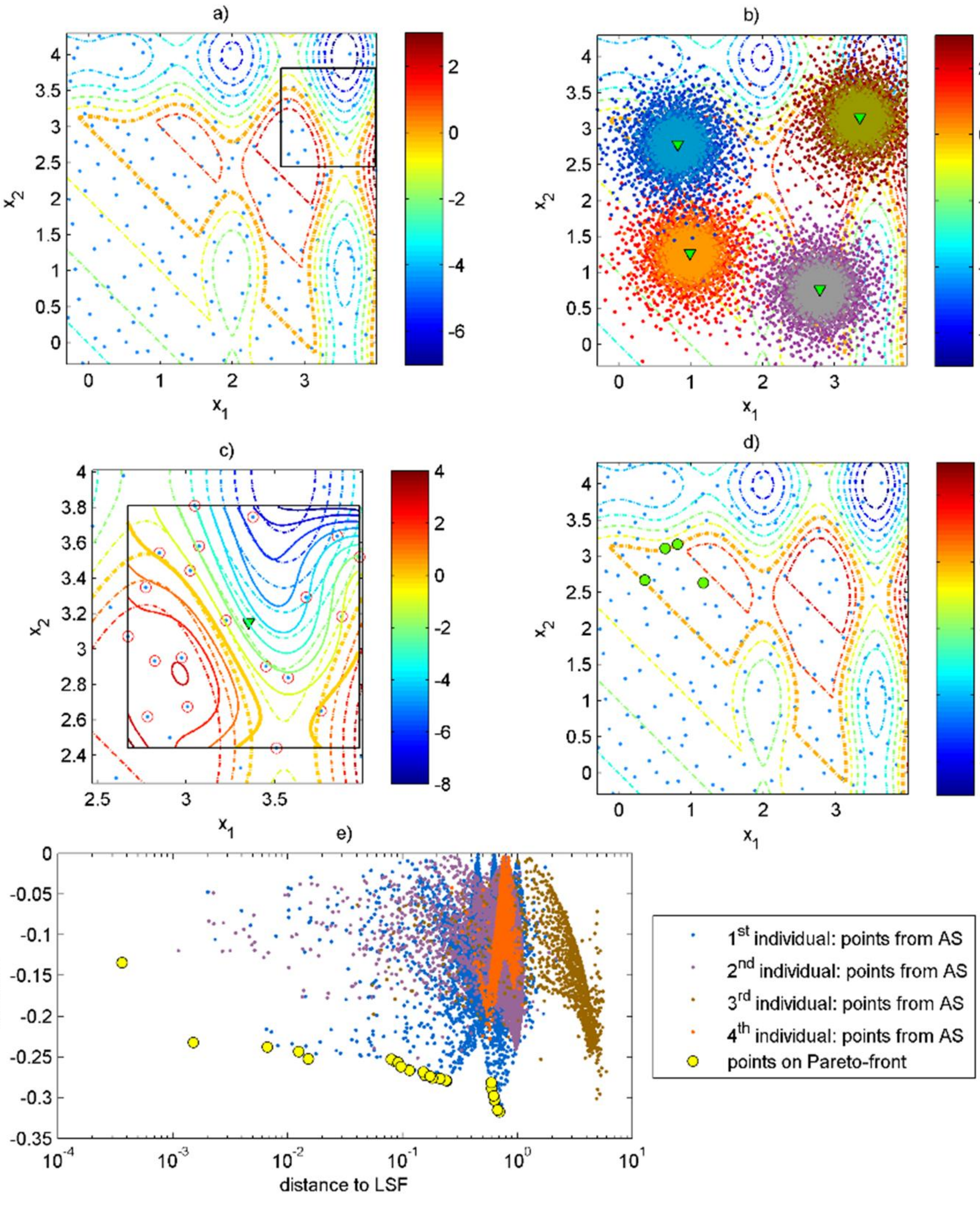
x1
x2
0
0.5
1
1.5
2
2.5
3
3.5
4
3
2
1
0
-1
-2
-3
-4
-5
-6
-7



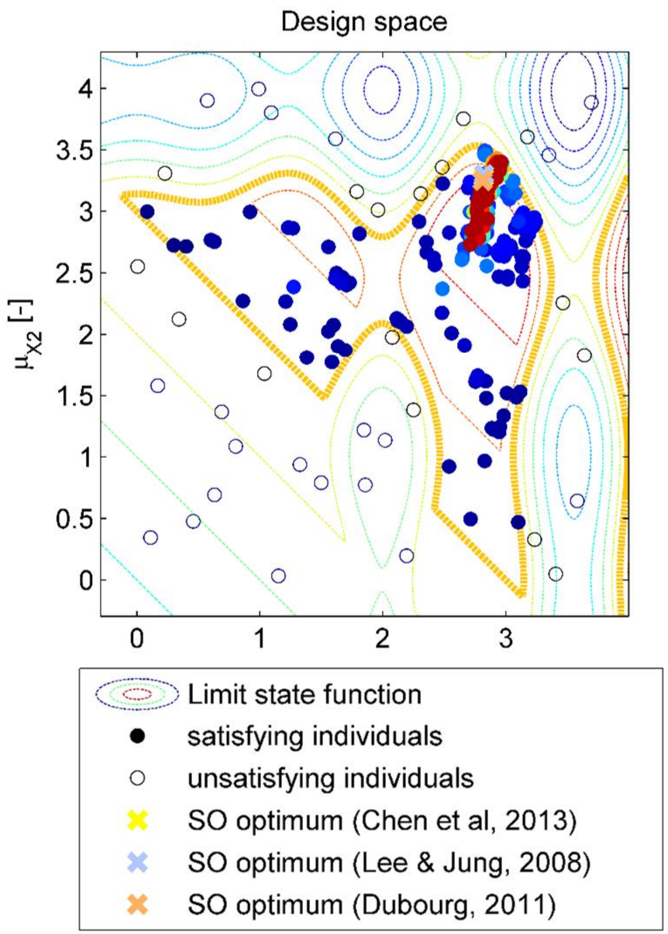
0
-0.05
-0.1
-0.15
-0.2
-0.25
-0.3
-0.35
-minimax
10^-4
10^-3
10^-2
10^-1
10^0
10^1
distance to LSF
1st individual: points from AS
2nd individual: points from AS
3rd individual: points from AS
4th individual: points from AS
points on Pareto-front

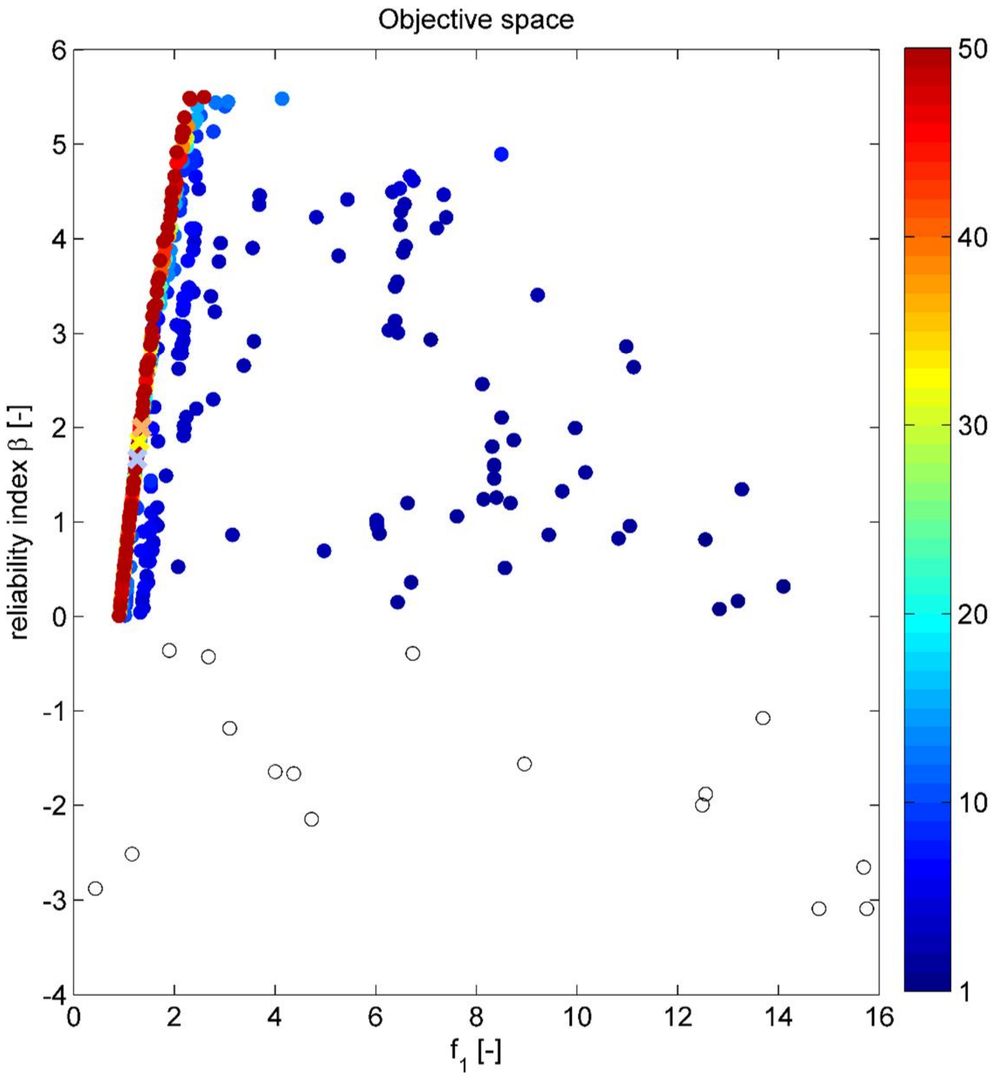


x1
x2
-0.5
0
0.5
1
1.5
2
2.5
3
3.5
4
3
2
1
0
-1
-2
-3
-4
-5
-6
-7

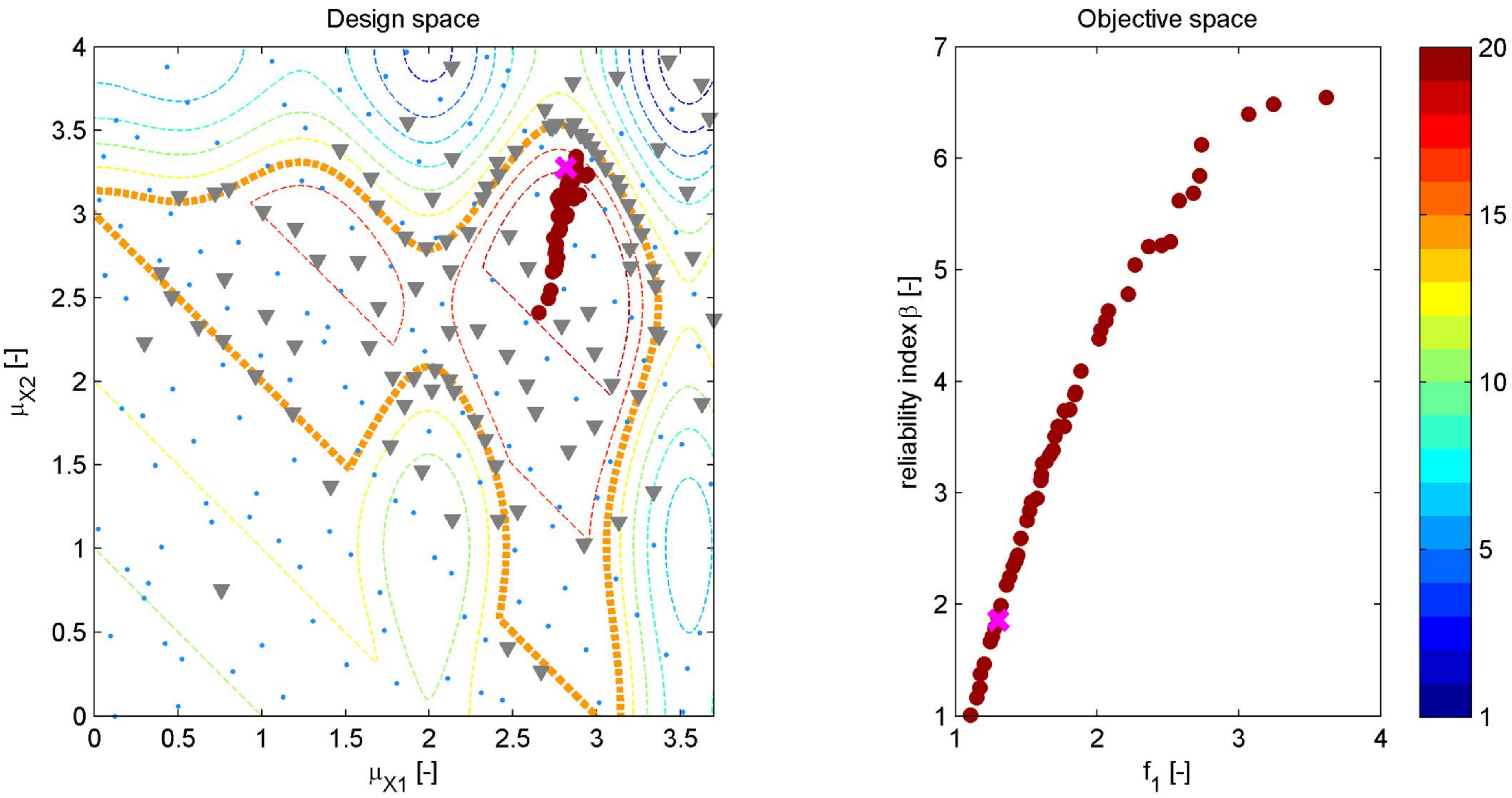

a)
b)
c)
d)
e)
x1
x2
-minimax
distance to LSF
1st individual: points from AS
2nd individual: points from AS
3rd individual: points from AS
4th individual: points from AS
points on Pareto-front

2D Benchmark: RBDO Convergence





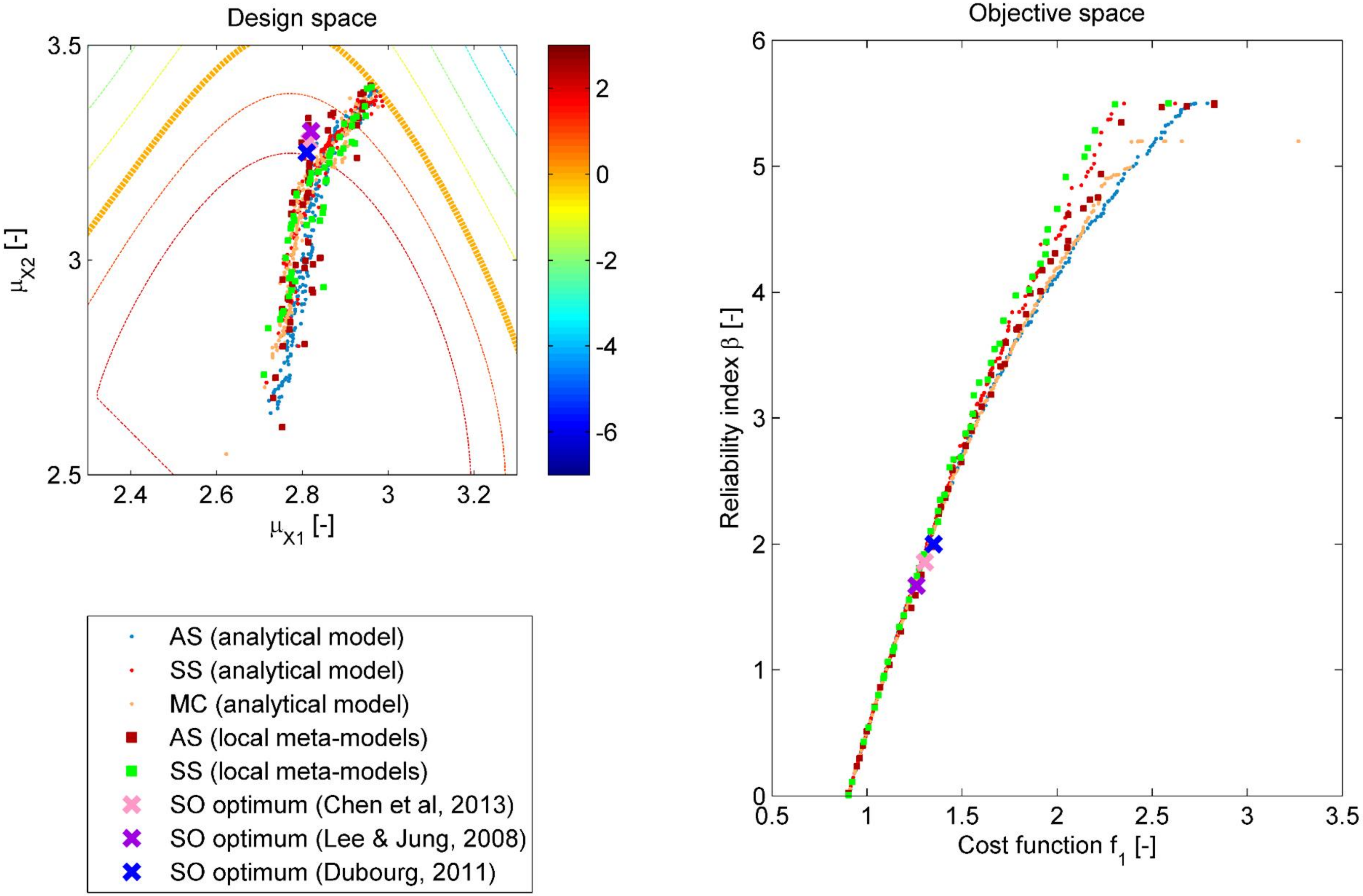
Final DoE and population



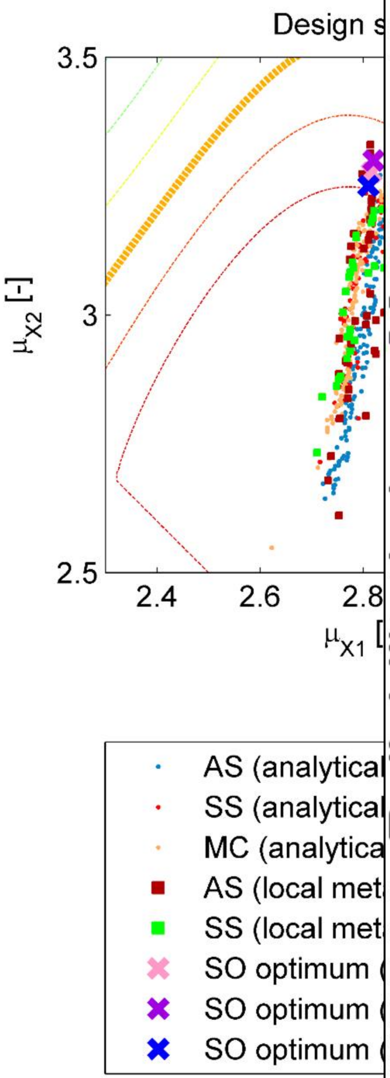
Computational demands for 2D problem

Number of primary DoE	200
Number of added samples to DoE during optimization	124
Number of analytical limit state function evaluation (FEA)	324
Number of objective function evaluations (and number of MM built for opt. purposes)	1,000
Number of meta-models built for DoE update purposes (and their evaluations)	407
Number of meta-model evaluations for optimization and reliability assessment purposes	7,943,168
Elapsed time	832 seconds

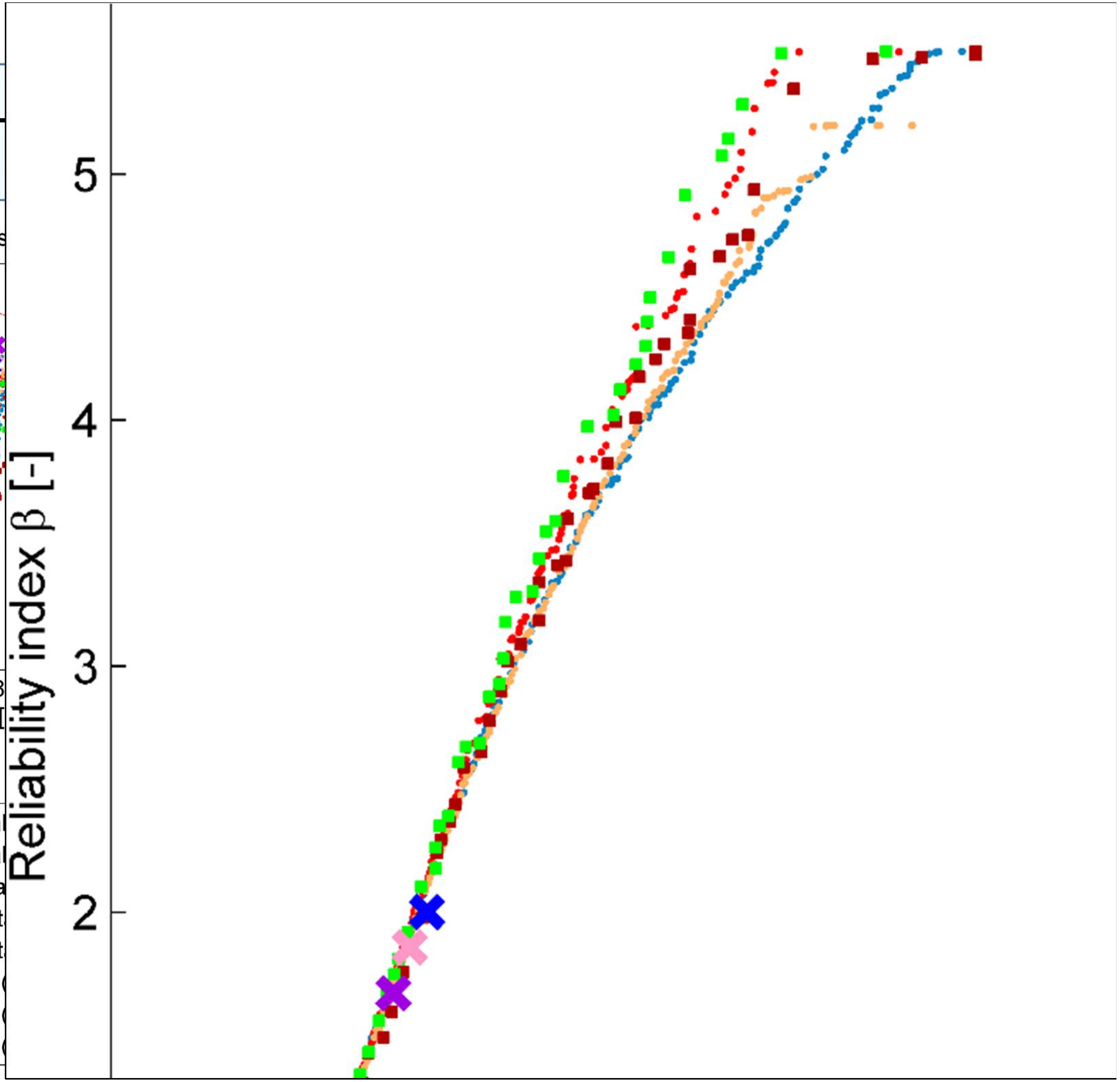
2D BENCHMARK: RBDO COMPARISON



2D Bench



Design s
3.5
3
2.5
2.4
2.6
2.8
μ_{X2} [-]
μ_{X1} [
AS (analytical
SS (analytical
MC (analytica
AS (local met
SS (local met
SO optimum (
SO optimum (
SO optimum (



5
4
3
2
Reliability index β [-]

CONCLUSIONS

- Multi-objective formulation of RBDO provides more information than a single-objective case for a decision maker.
- Several different techniques – crude Monte Carlo simulation, Subset simulation, Asymptotic sampling and First Order Reliability Method were compared within RBDO runs.
- Computational demands can be minimized by application of local meta-models
  - Computational demands for 10D benchmark (more than 1000 points):

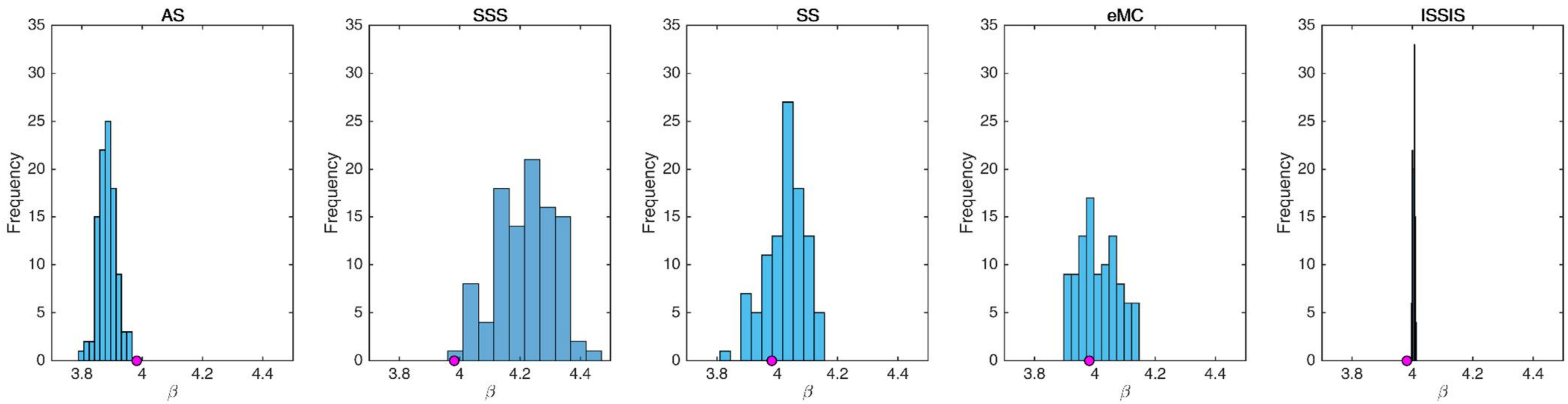
One global M-M	Local M-M
82.75 min	8.07 min

Current research

- New reliability assessment method based on Importance Sampling (ISSIS)

2D Benchmark:

[2.76199, 2.97517]



THANK YOU FOR YOUR ATTENTION.